Online Appendix for "The Rising Cost of Climate Change: Evidence from the Bond Market"

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Contents

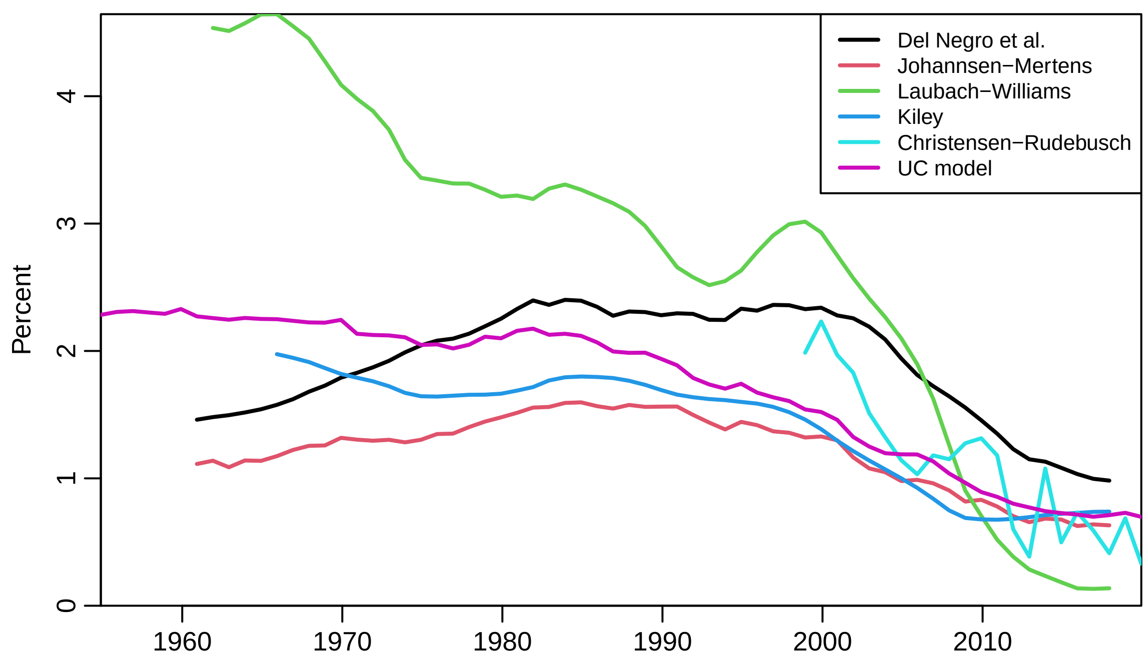
A Recent evidence on the decline in r_t^*

Given the importance of the equilibrium real interest rate, researchers have used various empirical methods from macroeconomics and finance to try to pin it down. For example, Laubach and Williams (2003, 2016) infer r_t^* by using the Kalman filter to distinguish the real interest rate trend and cycle with a simple macroecomic model and data on a nominal short-term interest rate, consumer price inflation, and the output gap. Johanssen and Mertens (2016) and Lubik and Matthes (2015) provide closely related r_t^* estimates from a similar filtering of the macroeconomic data. By contrast, Christensen and Rudebusch (2019) use a finance model to estimate r_t^* using only real bond yields measured from inflation-indexed debt—namely, U.S.

Treasury Inflation-Protected Securities (TIPS). These securities can provide a fairly direct reading on real yields since 1997 when the TIPS program was launched, and the arbitrage-free dynamic term structure model helps identify the underlying r_t^* in spite of the presence of potentially sizable liquidity and risk premia. Such a finance-based measure of r_t^* has several potential advantages relative to macro-based estimates, notably, it does not require the correct specification of output and inflation dynamics. Our new estimates of r_t^* in the next section are broadly in the spirit of finance-based estimates.

Along with this variety of methods, there are several somewhat different conceptual definitions of the equilibrium real rate in the literature. Some researchers focus on a short-run equilibrium rate, which represents the current value of the real rate that would be consistent with the economy at full employment and stable inflation. Others consider a very long-run empirical equilibrium rate defined as the real rate that would prevail in the infinite future, as calculated, for example, from a statistical trend-cycle decomposition of real rates. In practice, these different definitions appear to be closely related, and in many models they coincide, for example, as in Laubach and Williams (2003). For our purpose, the long-run trend is the relevant concept, since that is what matters for the term structure of social discount rates.

Figure A.1: Macro-finance estimates of the equilibrium real interest rate



Alternative published estimates of the equilibrium real interest rate, r_t^*. The estimates are smoothed/two-sided estimates of the state-space models with macroeconomic and financial variables in Del Negro et al. (2017), Johanssen and Mertens (2016), Laubach and Williams (2016), Kiley (2020) and Christensen and Rudebusch (2019). The series are quarterly from 1971:Q4 to 2018:Q1.

In Figure A.1, we show existing macro-finance estimates of r_t^* from a range of empirical studies. All of these estimates are consistent with our definition of r_t^* as the long-run trend component of the real short-term interest rate.

- Del Negro, Giannone, Giannoni and Tambalotti (2017)(DGGT) use a Bayesian framework to estimate a linear state-space model with common trends r_t^* and π_t^*. Their estimation uses five data series: observed price inflation, long-run inflation expectations, 3-month Treasury bill rate, 20-year Treasury yield, and long-run survey expectations of the 3-month Treasury yield.
- Johanssen and Mertens (2016) (JM) similarly estimate the long-run real rate trend from a time series model, but explicitly account for the zero lower bound on nominal rates and account for stochastic volatility.
- Laubach and Williams (2003, 2016) (LW) use a simple macroeconomic model and the Kalman filter to infer the *neutral* real interest rate, that is, the level of the real rate consistent with real output at potential and inflation at target.
- Kiley (2020) augments the Laubach-Williams model to account for changes in financial conditions. Both specifications assume that the neutral rate follows a martingale, so these r_t^* estimates are consistent with the long-run concept we employ.
- Christensen and Rudebusch (2019) estimate a dynamic term structure model for real (TIPS) yields, as discussed above. The estimation uses a Kalman filter, and r_t^* is estimated as the five-year-ahead five-year average of the expected future real short rate.

In addition, Figure A.1 also shows our own estimate of r_t^* described in the next section, which is obtained from an unobserved-components (UC) model for the real one-year U.S. Treasury bond yield (the baseline model of our empirical analysis of SDRs below).

It is evident from Figure A.1 that the various estimates of r_t^* have all declined substantially from the 1990s to recent years. The exact magnitude and pattern of the decline differs across models: These differences reflect the estimation and specification uncertainty that is a feature of statistical inference about long-run trends, and in particular about the equilibrium real rate (Laubach and Williams, 2016). But the overall pattern of a pronounced decline in the estimated path of r_t^* since the 1990s is quite consistent across all of the various specifications. Table A.1 summarizes the time profiles of these estimates. For each model, the table provides the average r_t^* during the 1990s and the 2010s and the difference between these two decadal averages. The bottom line in the table also provides the averages across the six estimates. All estimates show a decline across the decades, with a mean decline of 1-1/4 percentage points.

Table A.1: Estimates of the equilibrium real interest rate (r_t^*)

	1990s	2010s	Change
Del Negro et al. (2017)	2.3	1.1	-1.2
Johannsen and Mertens (2016)	1.4	0.7	-0.7
Laubach and Williams (2016)	2.8	0.3	-2.5
Kiley (2015)	1.6	0.7	-0.9
Christensen and Rudebusch (2019)	2.1	0.6	-1.5
UC model, 1y rate	1.7	0.7	-0.9
Mean	2.0	0.7	-1.3

Model estimates of r_t^* (in percent) during recent decades and the changes between these decadal averages (in percentage points). The "1990s" value is the average r_t^* from 1990 through 1999, except for the estimate of Christensen and Rudebusch (2019), where we report the average from 1998 (the first available observation) through 1999. The "2010s" value is the average r_t^* from 2010 through 2018 or 2019, depending on data availability. The "UC model, 1y rate" estimates are based on our own empirical unobserved-components model for the one-year real rate, described in Section 3 of the paper. Mean values—averaged across all six models—are shown at the bottom.

B Affine term structure of SDRs with time-varying r_t^*

As described in Section 2 of the paper, the equilibrium real interest rate, r_t^*, anchors the term structure of SDRs. In this appendix, we derive this result using a simple parametric model for the real short rate and the assumption of risk-neutrality. This specification provides a tractable affine dynamic term structure model for real interest rates.

We assume that r_t^* is a random walk without drift, that the cyclical component, $\tilde{r}_t$, follows a first-order autoregressive process, and that innovations are Gaussian:

$$r_t^* = r_{t-1}^* + u_t, \quad u_t \sim N(0, \sigma_u^2) \tag{1}$$

$$\tilde{r}_t = \phi \tilde{r}_{t-1} + v_t, \quad u_t \sim N(0, \sigma_v^2). \tag{2}$$

The assumption of risk-neutrality implies that the stochastic discount factor is $M_t = \exp(-r_t)$.

This model implies that the discount factor/bond price is exponentially affine in the two risk factors:

$$P_t^{(n)} = \exp(A_n + B_n r_t^* + C_n \tilde{r}_t), \tag{3}$$

where the affine loadings follow the recursions

$$A_{n+1} = A_n + \frac{1}{2}B_n^2\sigma_u^2 + \frac{1}{2}C_n^2\sigma_v^2 \tag{4}$$

$$B_{n+1} = -1 + B_n \tag{5}$$

$$C_{n+1} = -1 + \phi C_n, \tag{6}$$

with initial conditions $A_0 = B_0 = C_0 = 0$, and solutions $B_n = -n$ and $C_n = -(1-\phi^n)/(1-\phi)$. These results are easily derived by positing the structure in (3), using the bond price recursion $P_t^{(n+1)} = \exp(-r_t)E_t P_{t+1}^{(n)}$ and matching coefficients, similar to Ang and Piazzesi (2003). Yields are then given as $y_t^{(n)} = -A_n/n + r_t^* + (1-\phi^n)/[n(1-\phi)]\tilde{r}_t$. Forward rates are

$$f_t^{(n)} = -\frac{1}{2}B_n^2\sigma_u^2 - \frac{1}{2}C_n^2\sigma_v^2 + r_t^* + \phi^n\tilde{r}_t, \tag{7}$$

where the first two terms capture the convexity effects due to Jensen's inequality, and the last two terms reflect expectations. Note that in the special case where r_t is stationary, $\sigma_u^2 = 0$, the limiting forward rate (for $n \to \infty$) is a constant equal to $-\frac{1}{2(1-\phi)^2}\sigma_v^2 + r^*$, whereas if $\sigma_u^2 > 0$ the forward rate diverges to minus infinity. Equation (7) illustrates that short-term discount rates are affected by cyclical deviations from trend, $\tilde{r}_t$, that long-term rates are pushed down by convexity, and, crucially, that all discount rates are equally affected by r_t^*.

C A Ramsey rule with shifting trend consumption growth

The well-known Ramsey equation for the social discount rate, which arises in a simple intertemporal utility optimization setting, provides a common frame of reference in the literature on social discounting. We can extend the standard Ramsey formulation to show how accounting for structural economic change naturally gives rise to a social discount rate that includes a time-varying trend component r_t^*. With time-separable power utility, the usual intertemporal optimality condition can be rewritten to yield the following expression for the term structure of SDRs:

$$y_t^{(n)} = \delta + \frac{\eta}{n}E_t \log(c_{t+n}/c_t) - \frac{\eta^2}{2n}Var_t \log c_{t+n}, \tag{8}$$

where δ is the pure rate of time preference, η is the curvature parameter of the period-utility function, and c_t is consumption (e.g., Gollier, 2013, ch. 4). For constant consumption growth, we obtain the classic Ramsey equation, and for *iid* Gaussian consumption growth, equation (8) yields the extended Ramsey equation.[1] As a more general formulation, we specify consumption

[1]In both cases the term structure of SDRs is flat because there is no uncertainty about future values of r_t.

growth as

$$\Delta \log c_{t+1} = g_t^* + \tilde{g}_t + \varepsilon_{t+1},$$

with a randow-walk trend component, $g_t^* = g_{t-1}^* + \eta_t$, a stationary, mean-zero cyclical component, $\tilde{g}_t = \phi \tilde{g}_{t-1} + u_t$, and an innovation ε_{t+1}. The shocks η_t, u_t and ε_t are assumed to be *iid* and Gaussian. The crucial feature is that consumption growth has a shifting endpoint, g_t^*, instead of a constant long-run mean.

With this consumption growth process, the term structure of SDRs is

$$y_t^{(n)} = \delta + \eta g_t^* + \frac{\eta(1-\phi^n)}{n(1-\phi)}\tilde{g}_t - \frac{\eta^2}{2n} Var_t \log c_{t+n}, \tag{9}$$

where g_t^* is a level factor and $\tilde{g}_t$ is a slope factor (a high value of $\tilde{g}_t$ implies a downward slope). The final term reflects the convexity of the discount factor and of course declines in n, but its exact formulation and behavior are of second-order interest here.[2] The short-term rate and its trend are

$$r_t = y_t^{(1)} = \delta + \eta(g_t^* + \tilde{g}_t) - \frac{\eta^2}{2}\sigma_\varepsilon^2 \quad \text{and} \quad r_t^* = \delta + \eta g_t^* - \frac{\eta^2}{2}\sigma_\varepsilon^2.$$

That is, the real-rate trend r_t^* and the trend in consumption growth g_t^* are linearly connected. Shifts in the trend growth in aggregate consumption therefore translate directly into changes in the trend component of the real interest rate and consequently level shifts in the term structure of SDRs.

D Estimates of UC model

The priors of our UC model are specified as follows: For ϕ we use a relatively diffuse prior, specifically a standard normal distribution that is truncated to the interval from -1 to 1, so that the process for $\tilde{r}_t$ is stationary. For the variances σ_u^2 and σ_v^2 we use inverse-gamma prior distributions, which we denote as $IG(\alpha/2, \delta/2)$. For σ_v^2 the prior is diffuse with $\alpha = 6$ and $\delta = 4$ which implies a mean of 1 and a standard deviation of 1.

A key modeling choice is our prior for σ_u^2, which is very tight around a prior mean of 0.04. Specifically, for this prior $\alpha = 100$ and $\delta = 0.04(\alpha - 2)$, which implies a standard deviation of 0.006 around this mean. This prior forces the estimate of r_t^* to be quite smooth by anchoring its innovation variance at a low value, as in Del Negro et al. (2017) and Bauer and Rudebusch

[2] Specifically, the final term decreases rapidly with maturity n and is unbounded. It does not converge to a finite limit because consumption growth contains a unit root and thus $Var_t \log c_{t+n}$ is $O(n^2)$.

(2020). Under our prior, the change in r_t^* over 100 years has a variance of 4, and a standard deviation of 2 (percent).

Table D.1: Prior and posterior distributions for parameters of UC model

	Prior		Post. 1y rate		Post. 10y rate	
	Mean	SD	Mean	SD	Mean	SD
σ_u^2	0.040	0.006	0.041	0.006	0.042	0.006
σ_v^2	1.000	1.000	1.241	0.234	0.892	0.208
ϕ	0.000	0.540	0.763	0.117	0.642	0.216

Prior distributions for σ_u^2 and σ_v^2 are inverse-gamma, prior distribution for ϕ is standard normal, truncated so that $|\phi| < 1$. Posterior distributions are reported for two different model estimates, using either the one-year or ten-year real rate, with an annual interest rate sample that starts in 1954 (1y rate) or 1968 (10y rate) ends in 2019.

Table D.1 reports means and standard deviations for the priors and the posteriors of two estimated UC models, using the one-year and ten-year real interest rate series, respectively. For σ_u^2, the posterior distributions are very similar to the tight prior distribution. This raises the question how sensitive our estimation results are to this prior choice, and in particular how they would change with a more diffuse prior for σ_u^2. This question is answered in Table D.2, which reports the results of a prior-sensitivity analysis. In addition to our baseline prior, we estimate the models under three alternative, more diffuse prior specifications. Each alternative prior distribution maintains a mean for σ_u^2 of 0.04, but increases the dispersion by lowering α to 50, 20, and 10, respectively. Allowing for a more diffuse prior leads to more volatile estimates for r_t^*, as evident in higher posterior means for σ_u^2. As a result, the estimated decline in the real-rate trend between 1990 and 2019 becomes more pronounced. In other words, we find a substantial decline in r_t^* even under a very tight smoothness prior, and if we allow this prior to be more diffuse, we estimate an even larger decline.

E Estimates for long history of interest rates

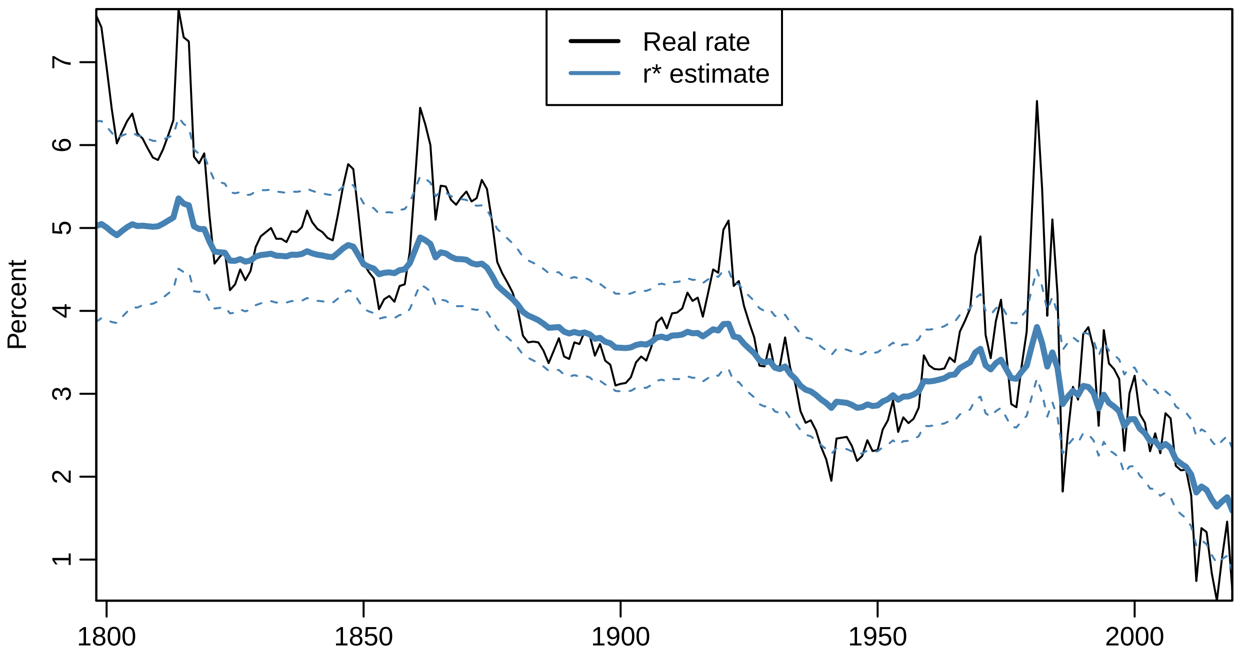
The paper shows estimates of r_t^* and implied SDR term structures for real interest rate series that start in the middle of the last century. But several previous papers have used substantially longer historical interest rate samples, such as Newell and Pizer (2003) who constructed a very long sample starting in 1798. Given that the goal of our analysis is to construct SDRs for very long horizons out to several hundreds of years, it would appear that one should also use as long a sample history as possible to estimate the SDR models.

Table D.2: Prior-sensitivity analysis for UC model

	Prior σ_u^2		Estimates 1y rate					Estimates 10y rate				
			Post. σ_u^2		r_t^*			Post. σ_u^2		r_t^*		
	Mean	SD	Mean	SD	1990	2019	Chg.	Mean	SD	1990	2019	Chg.
Baseline	0.04	0.006	0.041	0.006	1.9	0.7	-1.2	0.042	0.006	3.1	1.3	-1.8
Alt. 1	0.04	0.008	0.042	0.009	1.9	0.7	-1.2	0.044	0.010	3.1	1.2	-1.9
Alt. 2	0.04	0.014	0.046	0.017	1.9	0.7	-1.3	0.053	0.022	3.2	1.1	-2.2
Alt. 3	0.04	0.023	0.055	0.037	2.0	0.6	-1.4	0.077	0.050	3.4	0.9	-2.5

Estimation results for the UC model for four different inverse-gamma prior distributions for σ_u^2, including baseline specification with a tight prior and three alternative specifications with more diffuse priors. Estimates for 1y (10y) rate use an annual interest rate sample from 1954 (1968) to 2019. Columns labeled r_t^* report posterior mean estimates of the trend component for 1990 and 2019, as well as the change over this period.

Figure E.1: Estimate of r_t^* using UC model on long interest rate history



Real interest rate r_t and estimated equilibrium real rate, r_t^*. Real rate series is constructed as in Newell and Pizer (2003), using Treasury yields and an adjustment for expected inflation starting in 1955; updated to 2019 by Newell et al. (2020). Sample period: 1798-2019. r_t^* is the posterior mean from estimated UC model, with dashed lines showing 68% Bayesian credibility intervals.

Figure E.1 shows the Newell-Pizer interest rate series starting in 1798, and extended to 2019 by Newell et al. (2020).[3] Also shown is an estimate of r_t^* based on our UC model. Over the 19th century, this estimate of r_t^* has largely moved sideways until the 1990s, when a

[3]We thank Brian Prest for sharing the interest rate data series.

pronounced downward trend started and continued until the end of our sample. Over the last three decades of the sample, the estimated value of r_t^* has declined from about 3 percent to about 1 percent. These changes are very similar to those we estimate for the ten-year yield in our sample that starts in 1968, see Figure 1 in the paper.

Table E.1: Estimates of r_t^* using Newell-Pizer interest rate data

Model	1990	2019	Change
UC model	3.2	1.5	-1.7
AR model, break	4.3	2.3	-2.0
AR model, learning	3.6	3.0	-0.7

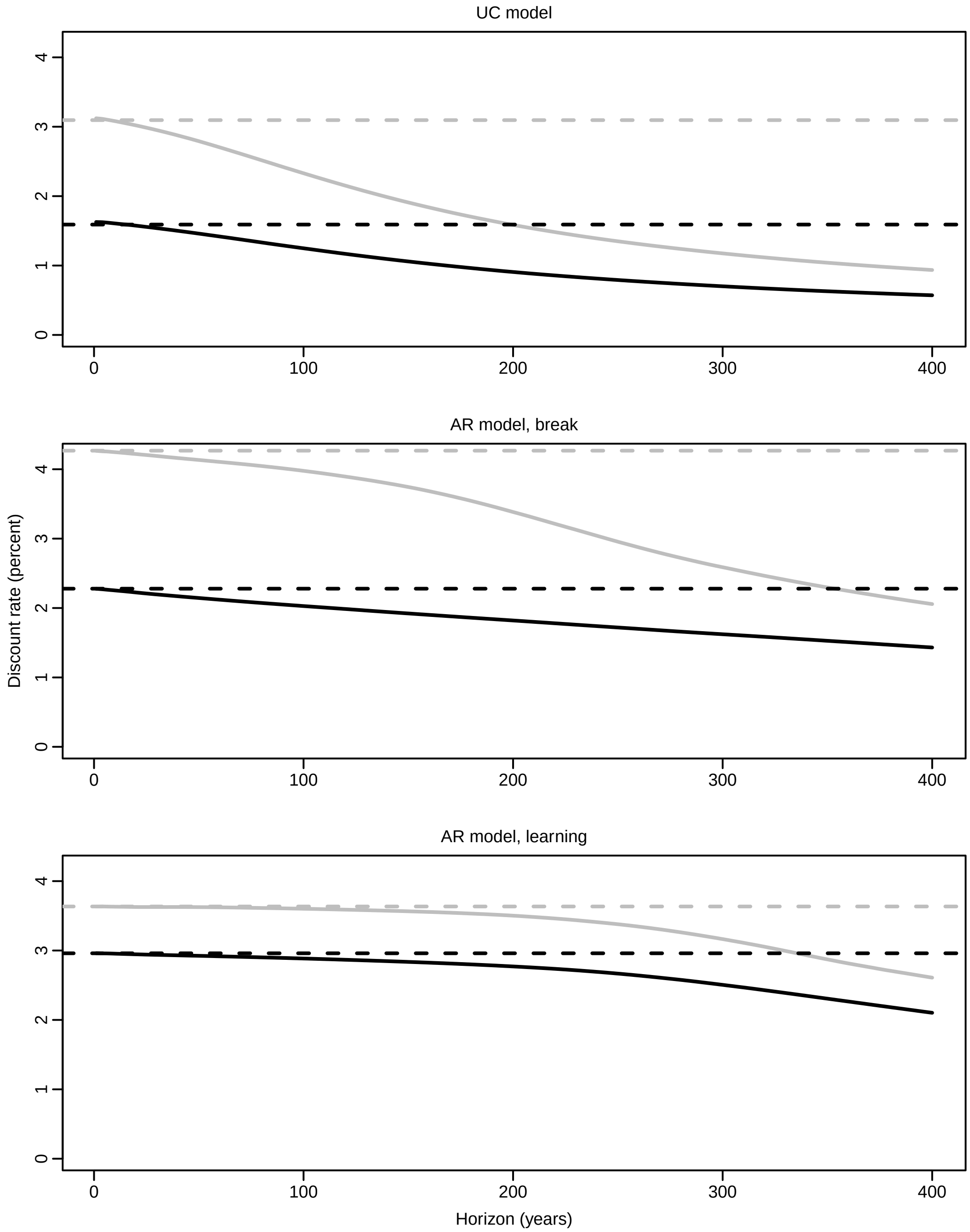
Model-based estimates of r_t^* using the long historical interest rate series from Newell and Pizer (2003) going back to 1798, updated to 2019 by Newell et al. (2020). The time series models are described in the text.

For our three different time series models, Table E.1 shows the values for r_t^* that we estimate from the Newell-Pizer data, as well as the change between 1990 and 2019.

Figure E.2 shows the implied term structures of SDRs. The term structures are calculated using simulations from the models as described in Section 4 of the paper, including a shadow-rate constraint on the real interest rate that ensures non-negativity. As before, the term structures are shown for values of r_t^* in 1990 and 2019. They look almost indistinguishable from those based on the ten-year yield (see Figure 2 in the paper).

Overall, using a much longer historical interest rate sample does not affect our qualitative conclusions. With a shifting trend as in our UC model, including additional interest rate data from the more distant past has almost no effect on estimated trends or the implied SDRs over the last few decades. For the AR models, the estimates are also quite similar for the Newell-Pizer data. These findings substantiate our baseline estimates using real rate series starting in the middle of the 20th century.

Figure E.2: Model-based term structures of SDRs for Newell-Pizer data



Term structure of social discount rates (SDRs) implied by alternative models estimated on the long historical interest rate series from Newell and Pizer (2003) going back to 1798, updated to 2019 by Newell et al. (2020). The time series models are described in the text. The red and blue lines are the term structures based on the real rate and estimated r_t^* in 1990 and 2020, respectively; dashed lines show estimates of r_t^* in those years.

F Method for calculating the social cost of carbon

The social cost of carbon at time t is defined as

$$SCC(t) = -\frac{\partial W}{\partial E(t)} \bigg/ \frac{\partial W}{\partial C(t)} \,, \tag{10}$$

where $C(t)$ is aggregate consumption, $E(t)$ represents emissions, and W is welfare; see for example Nordhaus (2017). Welfare in the DICE model is

$$W = \sum_{t=0}^{T} R^t U(c(t)) L(t), \tag{11}$$

where the discount factor is $R = \frac{1}{1+\rho}$ with rate of time preference ρ, period utility is $U(c) = \frac{c^{1-\alpha}}{1-\alpha}$, a function of per-capita consumption c, and population is $L(t)$. With these definitions, we can rewrite the SCC as follows

$$\begin{aligned} SCC(t) &= -\sum_{\tau=t}^{T} R^{\tau} \frac{\partial U(c(\tau))}{\partial E(t)} L(\tau) \bigg/ R^t \frac{\partial U(c(t))}{\partial C(t)} L(t) \\ &= -\sum_{\tau=t}^{T} R^{\tau-t} \frac{\partial U(c(\tau))}{\partial C(\tau)} \frac{\partial C(\tau)}{\partial E(t)} \frac{L(\tau)}{L(t)} \bigg/ \frac{\partial U(c(t))}{\partial C(t)} \\ &= -\sum_{\tau=t}^{T} R^{\tau-t} \left(\frac{c(\tau)}{c(t)} \right)^{-\alpha} \frac{\partial C(\tau)}{\partial E(t)}. \end{aligned}$$

The first equality holds because (i) only period utility in t is affected by a marginal change in consumption $C(t)$ and (ii) only utility from period t onward is affected by a marginal change in emissions $E(t)$. The second equality uses the fact that emissions affect utility by changing current and future consumption. The third equality substitutes the partial derivative $\partial U(c)/\partial C = c^{-\alpha}/L$. The end result is that, in the DICE model, the SCC is the present value of future marginal consumption damages $MD_{t+n} = -\frac{\partial C(t+n)}{\partial E(t)}$, which are known at t since there is no uncertainty in the model. The future damages are discounted using the consumption discount factor $R^n \left(\frac{c(t+n)}{c(t)}\right)^{-\alpha}$, which depends on pure time discounting and consumption growth between t and $t+n$, just like the discount rate in the classic Ramsey equation. We replace this discount factor by the empirical estimates of $P_t^{(n)}$ from our time series models to compute the value of the SCC in different years.

G Social cost of carbon from alternative DICE models

Here we report additional results for SCC calculations from two different versions of the DICE model. The approach to obtain a profile of marginal damages from increased GHG emissions is always the same, and follows Newell and Pizer (2003): We run the model with a one-period shock to exogenous CO_2 emissions in the baseline year, and calculate the consumption damages by comparing this model run to the results without a shock. We double-check that we can reproduce the model's internal SCC estimate using the model-implied path of the real interest rate and the marginal consumption damages we obtained in this manner.[4] Then we calculate estimates of the SCC using our own term structures of SDRs instead of the model's own internal discount rates.

Table G.1: Estimates of the SCC from alternative DICE models

Model	Change in r_t^*	1990	2019	Change
DICE-94 model (Newell and Pizer, 2003)				
UC model, 1y rate	-1.2	35.7	71.3	100 %
UC model, 10y rate	-1.8	19.3	60.2	212 %
AR model, break, 1y rate	-2.2	13.9	74.4	434 %
AR model, break, 10y rate	-1.9	7.4	27.9	277 %
AR model, learning, 1y rate	-1.5	12.8	40.9	218 %
AR model, learning, 10y rate	-1.5	6.5	19.1	193 %
DICE-2016 model (Nordhaus, 2017)				
UC model, 1y rate	-1.2	974.2	2458.3	152 %
UC model, 10y rate	-1.8	479.1	2040.9	326 %
AR model, break, 1y rate	-2.2	135.7	2242.6	1553 %
AR model, break, 10y rate	-1.9	65.2	425.2	552 %
AR model, learning, 1y rate	-1.5	119.7	741.0	519 %
AR model, learning, 10y rate	-1.5	50.0	236.6	374 %

Estimated social cost of carbon (SCC) for our six empirical SDR models, using marginal damage profiles from two different versions of the DICE climate-economy model. The estimated mean change in each model-based r_t^* from 1990 to 2019 is shown in percentage points. The SCC is calculated based on model-implied marginal consumption damages, in constant U.S. dollars, over the next 400 years resulting from one extra ton of CO_2 emissions in the base year. Estimated damages are obtained from two versions of the DICE model, (i) the DICE-94 model used in Newell and Pizer (2003), for which the damages are reported in constant 1989 dollars and the base year is 2000, and (ii) the DICE-2016 model, for which damages are in constant 2010 U.S. dollars and the base year is 2015. The columns "1990" and "2019" show SCC calculations using the term structures of social discount rates for 1990 and 2019 that are implied by each of our SDR time series models.

[4] As in Newell and Pizer (2003), we generally use the baseline or "no-controls" case in all of our model runs, instead of the optimized path of emissions. The only exception is for the updated DICE model of Hänsel et al. (2020), where we use the optimal mitigation path in order to stay as close as possible to their published SCC estimate.

For comparison with Newell and Pizer (2003), we also use the classic DICE-94 model. For this model, the future consumption damages are given in constant 1989 dollars, and like Newell and Pizer we use 2000 as the baseline year. The model yields an SCC estimate of \$5.29 (see Nordhaus, 1994). If we assume a constant 4% discount rate, then the SCC is \$5.70. This value is in line with conventional analyses that use constant discount rates and produce low estimates of climate change damages and support modest mitigation efforts (Newell and Pizer, 2003). Instead, if we assume a constant 2% discount rate, which is closer to the 1990 estimates of the equilibrium real interest rate, the social cost of carbon jumps to \$21.70, highlighting the importance of the level of the discount rate. Of course, a constant discount rate makes little sense in an uncertain world with the bond convexity/Weitzman effect ensuring that the term structure of SDRs is declining. The top panel of Table G.1 shows how the SCC changes for different term structures of SDRs. The UC model for the 1y rate shows that for 1990 the SCC is estimated to be \$35.70, substantially above the value for a constant 2% discount rate. This demonstrates the effect of a declining term structure of SDRs. For the DICE-94 model, the secular shift in the SDR term structure increases the estimated SCC by about 100-430 percent. While the absolute magnitude of the SCC is smaller for DICE-94, the relative changes are roughly in line with the estimates for the DICE-H model reported in Table 2 of the paper.

Our second alternative model is the baseline version of the DICE-2016 model, used in Nordhaus (2017), among others.[5] Just like for the DICE models we use in the main text, which are variants of this model, the SCC is reported in constant 2010 dollars and the baseline year is 2015. The model implies an SCC of \$31.20 using its own internal discount rates. For a constant 4% discount rate the SCC is \$36.10. While for a constant 2% discount rate the SCC increases to \$227. Moving to our own discount rates, the SCC increases substantially. This is due to two factors: First, even our 1990 risk-free discount rates are substantially lower than the model-internal discount rates, which Nordhaus calibrated to historical stock returns. Second, the temperature changes and climate damages from an increase in current emissions occur mostly in the (far) future, so that lower discount rates imply larger changes than for more front-loaded damages as in other versions of the DICE model, such as DICE-94 used by Newell and Pizer (2003) or the updated DICE model of Hänsel et al. (2020). Despite the much higher absolute magnitudes of the SCC estimates, the implications of a decline in r_t^* are qualitatively similar as for the other models (see Table 2 of the paper): The SCC increases substantially as a result of the downward shift in the term structures of discount rates.

Figure G.1 shows the different marginal damage profiles for all four DICE models we use for estimation of the SCC. For DICE-94, the future economic damages, also shown in Figure

[5]The GAMS code for this model is available at https://sites.google.com/site/williamdnordhaus/dice-rice.

6 of the working paper version of Newell and Pizer (2003), has a time path that increases steadily after the additional carbon release and reaches a maximum after about a century and then slowly declines over the remainder of the horizon. By contrast, for the DICE-2016 model the damage profile is *increasing*. This new damage function naturally leads to much higher SCC estimates when discount rates are low. Dietz et al. (2020) severely criticize the 2016 version of the DICE model exactly for producing too much global warming in the long run for a given level of carbon emissions. The adjustments they make to the model lead to substantially lower marginal damages and, given the same discount rates, SCC estimates. Compared to the DICE-2016 model, the present value of climate change damages is somewhat less sensitive to discount rates, as emphasized by Dietz et al. (2020). However, the resulting damage profile is still increasing with horizon. The modifications of Hänsel et al. (2020) more fundamentally alter the model-implied damages. As shown in Figure G.1, their model implies a hump-shaped profile with marginal damages that decline toward zero.[6]

In sum, across all four climate-economy models, which have vastly different estimates for the future economic damages from increasing GHG emissions, a downward shift in SDRs due to a lower r_t^* has very similar economic implications on the SCC. This finding underlines the important consequences of accounting for a lower new normal for interest rates in assessments of climate change damages.

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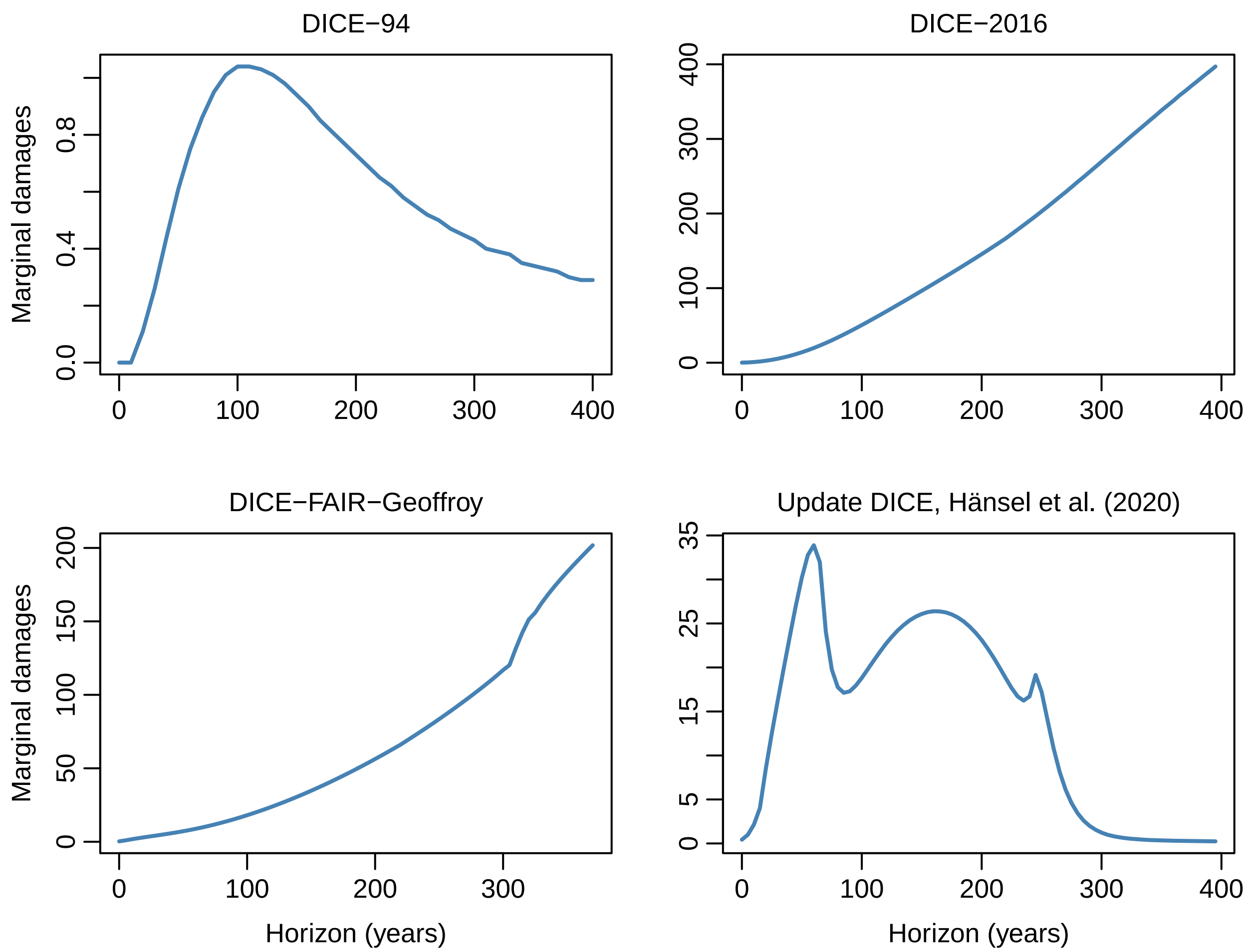
[6] This damage profile is obtained under an optimal mitigation policy.

Figure G.1: Marginal future damages (dollars per ton of CO_2) from alternative DICE models



Future damages, in constant US dollars, of emitting one additional metric ton of CO_2 in the baseline year. For the DICE-94 model, damages are in constant 1989 dollars, and the base year is 2000. For the DICE-2016 model and the other two models, damages are in constant 2010 dollars and the base year is 2015.

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