Interest Rate Skewness and Biased Beliefs*

Michael Bauer† and Mikhail Chernov‡

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Abstract

Conditional yield skewness is an important summary statistic of the state of the economy. It exhibits pronounced variation over the business cycle and with the stance of monetary policy, and a tight relationship with the slope of the yield curve. Most importantly, variation in yield skewness has substantial forecasting power for future bond excess returns, high-frequency interest rate changes around FOMC announcements, and consensus survey forecast errors for the ten-year Treasury yield. The COVID pandemic did not disrupt these relations: historically high skewness correctly anticipated the run-up in long-term Treasury yields starting in late 2020. The connection between skewness, survey forecast errors, excess returns, and departures of yields from normality is consistent with a theoretical framework where one of the agents has biased beliefs.

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†Unversität Hamburg; CESifo, michael.bauer@uni-hamburg.de.
‡Anderson School of Management, UCLA; NBER; CEPR, mikhail.chernov@anderson.ucla.edu.

1 Introduction

What predicts changes in interest rates? The literature has come a long way from the expectations hypothesis distinguishing between statistical, risk-adjusted, and subjective expectations. Regardless of the channel, the shape of the yield curve has emerged as a key source of information (e.g., Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005; Piazzesi, Salomao, and Schneider, 2015). The Global Financial Crisis of 2008/2009 led to unchartered territory for the yield curve, with short-term rates constrained by the zero lower bound (ZLB) and long-term rates affected by unconventional monetary policies. In such an environment, interest rate risk was generally perceived to be tilted to the upside, but the slope of the yield curve did not capture this. By contrast, interest rate skewness implied by Treasury options signaled substantial upside risk to yields, suggesting that implied skewness might be a useful forward looking measure to assess interest rate risk.

In this paper we argue that skewness is important for understanding yield dynamics and forecasting bond returns both during the ZLB period and outside of it, inclusive of the COVID pandemic. Because skewness measures the asymmetry in the distribution of future rate changes, it is a natural candidate for capturing changes in the balance of interest rate risks. That is in contrast to volatility, which captures market uncertainty about interest rates (e.g., MOVE or TYVIX indices). We study a model-free measure of the option-implied skewness of Treasury yields during the period from 1990 to 2021. We show that it indeed contains rich information about the outlook for interest rates throughout the whole sample.

Skewness exhibits substantial and persistent variation over time, which is related to the business cycle and the stance of monetary policy. Importantly, implied skewness contains significant additional predictive information, supporting the view that these changes in conditional skewness are indeed informative about the balance of interest rate risk. Specifically, implied skewness helps to anticipate bond excess returns, monetary policy surprises around FOMC announcements, and survey forecast errors for Treasury yields. We argue that our evidence is consistent with an environment where some economic agents have biased beliefs, or, equivalently, exhibit systematic expectational errors.

There is a dearth of evidence regarding this aspect of the distribution of interest rates. Indeed, many influential papers on non-normality of interest rates and its links to monetary policy implicitly or explicitly assume zero conditional skewness (e.g., Das, 2002, Johannes, 2004). Even the path-breaking work of Piazzesi (2001), which explicitly relates interest-rate jumps to FOMC announcements, treats the effects as symmetric in empirical work and focuses on the effects of uncertainty surrounding the announcements. There is a sound empirical reason for the extant perspective: the sample skewness of Treasury yield changes is close to zero.

Thus, to set the stage for our subsequent analysis, we start by investigating the time variation of conditional skewness of yields. We use two alternative measures of conditional skewness: The first is realized yield skewness computed from daily price data. The second is risk-neutral yield skewness as implied by Treasury option prices. The two measures are qualitatively similar in our case, their difference reflecting the skewness risk premium and measurement noise. For most of our empirical analysis, we focus on option-implied skewness, which has several advantages including its forward-looking nature, daily availability, and high statistical precision.

We document substantial and economically interesting time variation and persistence in conditional skewness over the past 30 years. In stark contrast to unconditional yield skewness, which is statistically indistinguishable from zero over this sample, conditional skewness indicates prolonged periods of both substantial upward and downward skew in the balance of interest rate risk. Option-implied skewness predicts future realized skewness, establishing formal statistical evidence for the time variation in conditional skewness. Aside from these cyclical fluctuations, yield skewness has generally drifted down over the course of our sample.

We then ask what the macroeconomic drivers of skewness in yields are. To this end, we assess the contemporaneous relationship of conditional skewness with financial and macroeconomic variables, including the shape of the yield curve and the stance of monetary policy. We find that yield skewness exhibits a close connection with a number of variables, the most pronounced being its positive correlation with the slope of the yield curve. Furthermore, skewness tends to be high when the Fed has recently eased the stance of monetary policy, and low when the Fed has tightened its policy stance.

Our main evidence is on the information content in option-implied skewness for future interest rates. We investigate whether the component of skewness that is orthogonal to the aforementioned variables carries useful information. To that end, we test if it can forecast key variables in bond markets and find that it does. Skewness exhibits highly significant predictive power for excess bond returns and FOMC surprises, which is robust to controlling for the shape of the yield curve and a wide range of other predictors. During the COVID episode skewness has correctly predicted a steep rise in 10-year yields from 0.5% in early 2020 to 1.5% in May 2021. Thus skewness appears to contain genuinely new information about the future evolution of interest rates.

Given that the documented predictability may arise from time-varying risk premiums or persistently biased beliefs, we next study the relation between survey forecasts and skewness. Using the Blue Chip Financial Forecasts (BCFF) and Survey of Professional Forecasters (SPF), we associate conditional yield skewness at the time of the surveys with the future consensus forecast error for the 10-year Treasury yield. We find a strong statistical relationship between skewness and this forecast error for all forecast horizons, which is robust to controlling for the shape of the yield curve. This evidence supports the view that time-varying differences between statistical and subjective expectations, or biased beliefs, play an important role for explaining the predictive power of skewness for interest rates. Skewness appears to be a proxy for the bias in beliefs about future interest rates.

That observation offers a clue about a possible economic mechanism behind the evidence. One needs a framework where some agents have biased beliefs. We adopt the heterogeneous beliefs two-agent framework by assuming that one agent knows the true distribution of the state, while the other one has erroneous beliefs. As a result, the usual measure of disagreement in these models becomes a measure of bias in beliefs.

The heterogeneous beliefs model of Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018) (EGHI) is particularly well suited for our purposes as it focuses on Treasury bonds. When investors differ in beliefs about future inflation, they take different positions in inflation-sensitive securities. In equilibrium, the investor who thinks inflation will be high buys inflation-linked and sells short nominal Treasury bonds, and an investor with the opposite view matches both positions on the other side to clear the markets. Ex ante, each investor expects to capture wealth from the other investor and, hence, both expect future consumption to be higher than without disagreement about inflation. Thus, the real interest rate depends on the dispersion in beliefs between the agents. Because of that dependence, the interest rate is non-normally distributed even if differences in beliefs have a normal distribution. Thus, time-variation in the dispersion in beliefs generates time-variation in yield skewness. Time-variation in bond risk premiums is driven by the differences in beliefs as well thereby linking skewness to expected excess bond returns.

Brooks, Katz, and Lustig (2020) document that FOMC announcement surprises cause a persistent drift in Treasury yields and provide an explanation based on non-rational expectations. Schmeling, Schrimpf, and Steffensen (2020) relate excess returns of financial instruments connected to short-term interest rates to survey forecast errors about future monetary policy. Conditional skewness, given its predictive power for both high-frequency announcement surprises and lower-frequency Treasury bond returns, captures a similar relationship. We find that skewness is directly connected to the expectation errors, which may be its source.

Giacoletti, Laursen, and Singleton (2021) (GLS) show that disagreement about future yields between 90th and 10th percentiles of BCFF predicts bond risk premiums. Further, the authors argue that their evidence is inconsistent with the heterogeneous beliefs mechanism. The standard channel affecting asset pricing is based on disagreement about fundamentals, while GLS document no relation between disagreemnts about inflation and yields. The mechanism that we advocate is distinct from the one explored in GLS. First, we show that skewness predicts bond excess returns together with the GLS measure of disagreement. Second, while the mathematics of biased beliefs and disagreement are very similar in a heterogeneous beliefs model, empirically they are different. We demonstrate, using consensus forecasts from both BCFF and SPF, that in contrast to the GLS evidence about disagreement, survey-based expectation errors about inflation are related to expectation errors about yields.

Broadly, our paper is related to the literature on interaction of monetary policy and bond markets pioneered by Balduzzi, Bertola, and Foresi (1997), Fleming and Remolona (2001), Ang and Piazzesi (2003), and Piazzesi (2005).

We forecast bond excess returns as in Cochrane and Piazzesi (2005), Ludvigson and Ng (2009), and Cieslak and Povala (2015). The role of monetary policy surprises for bond markets goes back to the work of Kuttner (2001) and Balduzzi, Elton, and Green (2001), and speaks to the recent work of Nakamura and Steinsson (2018). Errors in surveys about interest rates and the yield curve feature prominently in Buraschi, Piatti, and Whelan (2019), Cieslak (2018), and Piazzesi, Salomao, and Schneider (2015). Cieslak and Vissing-Jorgensen (2020) document the predictive power of downside risk for Fed easing actions, the "Fed put", which is related to our finding that asymmetric risks predict Fed policy actions.

In terms of using options to measure skewness, the methodology follows that of Bakshi and Madan (2000) and Neuberger (2012). Beber and Brandt (2006) and Trolle and Schwartz (2014) are primary empirical examples in fixed-income markets. The last paper is the closest to ours in terms of measurment: they apply the Neuberger (2012) approach to swaption prices and document time-variation in conditional skewness. The sample period is relatively short, 2002-2009, and the authors do not relate skewness to bond returns or survey-based forecasts of yields.

Hattori, Schrimpf, and Sushko (2016) demonstrate that unconventional monetary policy has reduced option-implied tail risks for the stock and bond markets. Mertens and Williams (2020) use option-implied distributions during the ZLB period to distinguish between the constrained and unconstrained monetary policy equlibria. Li (2020) documents the connection between option-implied Treasury skewness and recessions between 2000 and 2018.

The early work on heterogeneous beliefs is Harrison and Kreps (1978) and Detemple and Murthy (1994). Heterogeneous beliefs-based asset pricing applications are reviewed in Basak (2005). Xiong and Yan (2010) is the first application of heterogeneous beliefs to Treasury bonds. Our contribution to this literature is to point out a link between dispersion in beliefs and non-normality of yields, both in theory and in the data.

2 Time variation in interest rate skewness

Interest rate skewness captures the degree of asymmetry in the probability distribution of changes in interest rates. Given that average interest rate changes are close to zero, positive skewness indicates that large rate increases are more likely than large rate declines, which implies that the balance of risk is tilted to the upside, and vice versa.

Unconditional interest rate skewness—the sample skewness of Treasury yield changes over long periods of time—has been essentially zero. In this section we document that this contrasts with pronounced shifts and large cyclical swings over the last three decades in conditional skewness, measured either as realized skewness (using short rolling windows of Treasury yield changes) or option-implied skewness (using model-free moments implied by Treasury options). The statistical evidence for this time variation is that option-implied skewness strongly predicts realized skewness.

2.1 Data

The data we use in this analysis are Treasury yields, as well as Treasury futures and options. Our Treasury yields are the daily smoothed Treasury yield curves from Gürkaynak, Sack, and Wright (2007). When we need a monthly data frequency we take monthly averages. Most of our analysis focuses on the 10-year yield.

The Treasury derivative prices are from CME group. In particular, we use end-of-day prices of the 10-year T-Note futures contract, and options written on this contract. Both types of contracts are among the most actively traded Treasury derivatives, with high liquidity (in terms of open interest and volume) and long available price histories. The deliverable maturities for this futures contract are between 6.5 and 10 years. Changes in futures prices are closely associated with negative yield changes in the cheapest-to-deliver (CTD) Treasury security. Because skewness is scale invariant, we can take the negative of the skewness of futures price changes as a measure of skewness of yield changes for the CTD bond. Details are in Appendix A.1.

Our sample period is from the beginning of January 1990 to the end of May 2021. The starting date is dictated by the availability of options data allowing us to consistently calculate option-based moments using prices across many contracts/strikes. While the historical Treasury options data starts in May 1985, there are only few contracts and prices available during the early years.

2.2 Sample statistics and unconditional skewness

The top panel of Table 1 reports summary statistics for quarterly changes (using the last month of the quarter) in the 10-year yield, including the mean, median, variance and third central moment. We report the statistics for the full sample and the first and second half of the sample. In addition to sample statistics, we also calculate 90%-confidence intervals using a simple bootstrap, since yields are highly persistent and the serial correlation of their changes is close to zero. The mean and median are negative and, like the variance, have changed little between the first and second sub-sample. By contrast, the third sample moment of quarterly yield changes shifted: over the first half of the sample, the third moments is zero, while over the second half it has turned negative.

The middle and bottom panels of Table 1 report the sample skewness coefficient for yield changes and (negative) futures price changes, respectively, and we report skewness for different frequences, ranging from one-month to twelve-month changes. The results for bond yields and futures are similar: For the full sample, sample skewness of interest rate changes is statistically close to zero. Its value in split samples depends on the frequency, but it is typically higher and positive over the first half of the sample and negative over the second half of the sample. The magnitude ranges between -1 and 0.5, depending on the specific sample and frequency, is comparable to the skewness estimates for foreign currency and equity index returns reported in the literature (e.g., Chernov, Graveline, and Zviadadze, 2018, Table 1).

Thus, while the mean and variance of yield changes have not changed, the shape of the asymmetry has shifted noticeably. While the skew of the distribution generally appears slightly positive from 1990 to 2004, it has shifted significantly negative for the period from 2005 to 2021. This empirical pattern suggests that an unconditional, full-sample perspective on skewness may miss interesting features of the distribution of interest rates. Therefore we next turn to conditional yield skewness.

2.3 Realized and implied skewness

In order to measure yield skewness at a more granular level, and to understand time variation, we follow the literature on skewness in stock returns and calculate realized and option-implied measures of skewness. We use daily changes in prices and implied volatility for Treasury futures to compute realized skewness (RSK) at the monthly frequency (Neuberger, 2012, Equation 5). Again, the negative of futures price skewness corresponds to yield skewness.

Figure 1A plots this time series of RSK, as well as a 12-month moving average. Monthly realized yield skewness is volatile and on average close to zero, but exhibits some persistence and pronounced time variation. During three episodes skewness was markedly negative: the dot-com bubble 1998-2000, the financial crisis of 2007-2009, and the period since 2015 when the Fed lifted its policy rate off the ZLB. Skewness declines sharply in the wake of the COVID-19 pandemic in early 2020 but then reaches historical high level in the wake of global fiscal and monetary stimulus.

Realized skewness allows us to gauge time variation but it is noisy, available only at lower frequencies, and backward-looking. We now turn to option-implied skewness for future Treasury yields. This implied skewness (ISK) measures the conditional, risk-neutral skewness of future yield changes, as reflected in options on Treasury futures. Details of how we construct ISK are in Appendix A.1. The negative of implied skewness for Treasury futures price changes corresponds to skewness of Treasury yield changes. On every trading day we calculate ISK for each futures contract expiration. For most of our analysis, we then use ISK for the most active option contract, namely the shortest quarterly contract, which has a maturity between about 1 and 3 months.

Figure 1B shows a time series of implied skewness that is linearly interpolated to a constant horizon of 2.4 months (the average horizon of all option contracts). This daily series is highly persistent with a first-order autocorrelation of 0.95. Over the full sample, the average level of ISK is positive, with a mean of 0.10 that is significantly different from zero, and a standard deviation of 0.30. But this average level of skewness masks substantial variation in risk perceptions about for future Treasury yields. Similar to realized skewness, ISK has exhibited a general downward trend as well as pronounced cyclical swings over the course of our sample. Particularly striking is the behavior during the first ZLB episode, when the Fed's policy rate was near zero. Between 2009 and 2014, ISK averaged 0.35, while outside of this period the average was only 0.04. Before liftoff from the ZLB in 2015 skewness shifted markedly negative, and it averaged -0.21 between 2015 and the end of the sample. As is the case with RSK, the COVID stumulus has changed that trend with ISK reaching levels of around 1.0 in late 2020.

More formal statistical analysis is helpful to better understand the time variation that is visually evident in these figures. Specifically, we want to test whether time variation in option-implied skewness is statistically and economically significant. One straightforward way to do so is to assess whether it predicts realized skewness. In Table 2 we present results for various regression specifications, predicting monthly realized skewness with its own lag, option-implied skewness or the shape of the yield curve, all measured at the end of the previous month. To guard against serial correlation due to the persistence of RSK we report Newey-West standard errors (with automatic bandwidth selection). RSK exhibits significant autocorrelation, but lagged values of ISK have even stronger predictive power than lagged values of RSK itself. In a regression that includes both predictors, both are strongly significant (see column 3). The slope of the yield curve has some explanatory power for future RSK, but once we include ISK this is driven out and the information in the yield curve becomes irrelevant. In sum, there is strong evidence for time variation in the conditional expectation of RSK, as captured by ISK. This establishes ISK as a useful forward-looking measure of conditional yield skewness.

3 Macro-finance drivers of yield skewness

The previous section documents time variation in conditional skewness of yields. We now investigate which financial and macroeconomic drivers can potentially explain the time variation in conditional yield skewness. This analysis will lead us closer to the understanding of skewness' informational content.

It turns out that skewness exhibits a statistically and economically significant relationship with the slope of the yield curve and stance of monetary policy. Periods when the slope is high (low) or when the Fed has been easing (tightening) the stance of monetary policy are characterized by high (low) option-implied skewness. The orthogonal component of skewness still exhibits interesting cyclical variation, which our analysis in Section 4 shows to contain substantial additional predictive information. We find little evidence for a mechanical effect of the ZLB on the level of skewness, supporting the view that the importance of skewness is not limited to the ZLB period alone.

3.1 Skewness and the yield curve

To provide a visual impression of the relationship between interest rates and skewness, Figure 2 plots annual moving averages of implied skewness with the two-year and ten-year Treasury yields. In addition, the figure also shades monetary policy easing and tightening cycles, which we identify based on changes in the federal funds rate, since shifts in monetary policy are a key driver of shifts in the yield curve (Piazzesi, 2005).

Figure 2 reveals several patterns. Skewness tends to increase when the yield curve is steepening, in particular during the episodes in 2002-2003 and 2008-2013. Thus, the term spread appears to be related to skewness. In addition, there is also a relationship between skewness and the level of the yield curve, but it is more complicated and appears to depend on the frequency. On the one hand, there is a general downward trend in skewness and yields over the course of the sample. But on the other hand, the level of yields and conditional skewness seem to move in opposite directions at business cycle frequency.

We formalize the evidence by a series of regressions reported in Table 3. In all specifications, the dependent variable is conditional implied skewness, ISK. We use monthly averages of skewness and interest rates. Due to the high persistence of ISK, robust standard errors are calculated using the Newey-West estimator (with automatic bandwidth selection), and they are reported in parentheses.

The relationship between RSK and ISK is the natural baseline and starting point for an analysis of the drivers of implied skewness. Column 1 shows the corresponding regression, and the estimates indicate a very strong contemporaneous relationship between the two series. The next column reports estimates for a regression on the level and slope of the yield curve. These are calculated as the first two principal components of yields from one to ten years maturity, normalized such that high level/slope are associated with high yields/an upward sloping yield curve. The numbers confirm that the slope is important for skewness: an upward-sloping yield curve is associated with high skewness. Level, by contrast, does not have a statistically significant relationship with skewness. By including RSK, level and slope together we can account for more than half of the variation in ISK, with an R^2 of 0.50.

The two subsequent columns incorporate the interaction between level and slope, which adds substantial explanatory power. The slope has a much stronger relationship with skewness when the level of yields is low than when it is high, a pattern to some extent driven by the ZLB period when slope and level were both historically low and skewness skyrocketed. According to the specification in column 5, we can explain 54% of the variation in skewness just based on the yield curve—including the level-slope interaction—and RSK. Thus, variation in yield skewness is closely related to variation in yields themselves.

Figure 1C plots the residual from a regression of ISK on level, slope and a level-slope interaction, that is, from the regression specification in column 4 of Table 3. This residual exhibits neither the downward trend nor the pronounced negative level shift after 2014. Accounting for the shifts in the yield curve also dampens the large cyclical swings, compared to the original series. However, this unexplained portion still has interesting cyclical variation, which we show below to contain substantial relevant information about future yields.

3.2 Skewness and the ZLB

Section 2 documented that implied skewness was significantly higher during the ZLB period beginning in 2008. In light of this pattern, one might hypothesize that skewness generally tends to be high when interest rates are low, i.e., close to the ZLB. In particular, there could be a mechanical reason for positive skewness in the sense that the ZLB does not allow a long left tail and thus necessarily makes the right tail comparatively longer. If this mechanism was a key driver of conditional skewness, then it would suggest that skewness does not contain much forward-looking information about the direction of interest rate risk. But our results instead indicate that proximity to the ZLB itself is not an important factor explaining variation in conditional skewness.

First, if conditional yield skewness depended on the proximity of yields to the ZLB, then this should imply a negative relationship between skewness and the level of the yield curve. By contrast, the estimates in Table 3 suggest either a non-existent or positive relationship, depending on the specification. Furthermore, skewness is time-varying and sign-switching in our sample prior to 2008, without any apparent time trend despite the secular downward trend in interest rates (Bauer and Rudebusch, 2020). For example, yields were lower in 2016 than during most of the first ZLB period, yet skewness was mainly negative in 2016.

Second, skewness behaved very differently during the two ZLB episodes in our sample. Skewness turned positive when the ZLB was reached in 2008 and remained mainly positive for several years, but then switched to negative in 2014, more than a year before the Fed lifted its policy rate off the ZLB. During the most recent period, skewness remained negative for several months after the ZLB was reached, but then turned positive in mid-2020 before long-term Treasury yields commenced a pronounced increase, as discussed in Section 4.4.

Third, a closer investigation of the entire option-implied density of future yields during the two ZLB episodes shows no evidence of a mechanical ZLB explanation of positive skewness. Figure 3 shows, for two different dates, the implied densities for yields at the option expiration date, obtained from (i) the bond derivative prices,

(ii) a normal-inverse-gamma distribution that matches the first four option-implied moments (Eriksson, Ghysels, and Wang, 2009), (iii) an approximate mapping from bond price changes to yield changes (see Appendix A.1), and (iv) the current CTD bond yield.

The first date is December 31, 2012, a day with a particularly low yield level (1.14 percent) and a high level of skewness (0.8). The density pertains to the yield level on February 22, 2013, the expiration date. Even for this extreme example of low yields and high skewness during this episode, the 1st percentile of the distribution is comfortably above the ZLB, at 0.6 percent. This suggests that the left tail is not thinner because it is cut off by the ZLB, but instead because investors perceived an upward tilted balance of risk and large right tail.[1]

The second date is June 16, 2020, a day with extremely low yields and *negative* skewness, which was not uncommon during this episode. For this distribution, pertaining to the yield level on August 21, the 1st percentile is deeply negative, at -0.6 percent, suggesting that the ZLB does not eliminate left tails and mechanically lead to positive skewness, at least not during this episode.

Overall, this evidence suggests that the high level of skewness during the first ZLB episode was not mechanically driven by the proximity to the ZLB itself. Instead, other factors affected risk perceptions during this episode, such as unconventional monetary policies implemented by the Fed and the investor view that long-term yields might return to higher levels if the impact of such policies proved transitory. Our results below on the predictive power of skewness will further support the view that variation in skewness reflects a changing outlook on interest rate risk and not a mechanical impact of the ZLB.

3.3 Skewness and the monetary policy cycle

Figure 2 shows that skewness tends to increase during or after monetary easing cycles, most prominently during the easing after the 2000 dot-com bust and during the

[1]Even a counterfactual distribution with an equally large negative skewness, at -0.8, still has a 1st percentile quite a bit above the ZLB, at 0.3 percent.

Great recession and ZLB period. Vice versa, tightening episodes coincide with or precede episodes of falling skewness. We now dig deeper into the relationship between skewness and monetary policy cycles.

To this end, we estimate regressions that include indicator variables for monetary easing and tightening cycles. Cross-correlations reveal that one-year lags of the indicator variables for easing and tightening episodes have the strongest correlation with skewness, so we include these lags instead of contemporaneous indicators in our regressions. Columns 6 and 7 of Table 3 show the results.

Conditional skewness has a statistically strong and economically intuitive relationship with the monetary policy cycle. A regression of ISK on the cyclical indicators, shown in column 6, demonstrates their substantial explanatory power, with an R^2 of 0.31. A third of the variation in conditional yield skewness is explained by the monetary policy cycle. Skewness is high during and soon after monetary easing cycles. Intuitively, these are episodes of upward-tilted interest risk because the Fed has been lowering rates and the next monetary tightening cycle is likely to begin soon. Vice versa, skewness is low during and early after monetary tightening cycles, periods where investors are turning their attention to downward risk to interest rates. Thus, skewness appears to capture the changing balance of interest rate risk over the business cycle.[2]

The relationship of skewness with the monetary policy cycle is so strong to even drive out the relationship with the yield curve. When we add these indicators to a regression with RSK and the yield-curve variables, the slope becomes insignificant, as shown in column 7 of Table 3. This finding may be understood in light of the fact that the slope of the yield curve mainly reflects the stance of monetary policy (Rudebusch and Wu, 2008) and our two cylical indicators apparently provide a more nuanced measure of the monetary policy cycle.

Overall, we find very strong contemporaneous statistical relationships, so that the

[2]We have also found evidence for the cyclical behavior of skewness using various business cycle variables, including NBER recession dummies, industrial production grpwth, the output gap, the Chicago Fed National Activity Index, among others. The two cyclical indicators in Table 3 are so closely related to skewness that when we include them, other macroeconomic variables generally become insignificant. We omit these results for the sake of brevity.

shape of the yield curve together with indicators of the state of the monetary policy cycle (in addition to past realized skewness) can explain a large share of the variation in conditional yield skewness. When the yield curve is upward-sloping or the Fed has been easing its policy stance then implied skewness tends to be high, and vice versa. Given these strong contemporaeous correlations, it is important to control for this type of information in our predictive analysis, which shows that skewness is more informative about the future evolution of yields than any of these indicators.

4 The information in conditional skewness

Our evidence so far has established the cyclical variation in skewness and linked it to economic driving forces and the business cycle. We now turn to the question whether yield skewness contains useful forward-looking information for interest rates. Consider the link between expectations about bond returns and risk premiums:

$$E_t(RX^{(n)}_{t,t+1}) = E_t(RX^{(n)}_{t,t+1}) - E^s_t(RX^{(n)}_{t,t+1}) - \frac{Cov^s_t(M_{t,t+1}, RX^{(n)}_{t,t+1})}{E^s_t(M_{t,t+1})}, \qquad (1)$$

where $RX^{(n)}_{t,t+1} = P^{(n-1)}_{t+1}/P^{(n)}_t - 1/P^{(1)}_t$ is the excess gross return on $n-$period bond with a price $P^{(n)}_t$, $M_{t,t+1}$ is the stochastic discount factor (SDF), and superscript s refers to subjective probability.[3] This representation is helpful because it demonstrates that predictability of the future (excess) returns could be due either to variation in risk premiums or a time-varying bias in beliefs, or both. Variation in risk premiums has traditionally been the common explanation of empirical results documenting predictability of bond returns or interest rate changes. Recent work, however, has emphasized the possibility that such empirical correlations could be due to the failure of the often implicit assumption of full information rational expectations (FIRE), i.e., to changing biases in beliefs captured by the first two terms in equation (1) (e.g., Bauer and Swanson, 2020, Bacchetta, Mertens, and van Wincoop, 2009,

[3] Excess gross returns allow for the cleanest decomposition, while our analysis below in Section 4.1 shows empirical results for excess log returns, as is common in this literature. The two are very similar in the data, and our empirical results are essentially unchanged if we use excess gross returns.

Buraschi, Piatti, and Whelan, 2019, Cieslak, 2018, Piazzesi, Salomao, and Schneider, 2015).

This representation guides our empirical work. We first establish whether skewness predicts bond returns. We implement that analysis in two different ways: conventional predictive regressions for monthly excess returns on Treasury bonds, similar to Cochrane and Piazzesi (2005) and many others, and an analysis of monetary surprises at daily frequency.[4] Next, we disentangle the source of predictability by using consensus survey forecasts as a proxy for subjective expectations.

4.1 Bond excess returns

We work with monthly data, and follow common practice by using end-of-month interest rates. We use a one-quarter holding period for bond returns, given that ISK is based on derivative contracts with expiration of about 1-4 months in the future, and we calculate excess returns over the three-month T-bill rate as the risk-free short rate. We obtain log excess returns, $rx^{(n)}_{t,t,+3}$, in the usual way for all bonds with maturities from one through ten years. As in Cieslak and Povala (2015), we scale them by maturity so that all excess returns have the same duration.

Our predictive regresisons are of the form

$$\overline{rx}_{t,t+3} = \beta' X_t + \varepsilon_{t,t+3}, \tag{2}$$

where $\overline{rx}_{t,t+3} = \frac{1}{10}\sum_{n=1}^{10} rx^{(n)}_{t,t+3}/n$ is the weighted average log excess return across all maturities, X_t is a vector of predictors observable at the end of month t, and $\varepsilon_{t,t+3}$ is the serially correlated prediction error. The predictors in X_t always include, besides a constant, at least the level, slope and curvature of the yield curve, calculated as the first three principal components of yields, since a natural null hypothesis is that the current yield curve reflects all the information that is relevant for expectations of future interest rates (Duffee, 2011). For more reliable statistical inference in this

[4]Analysis of daily interest changes can be related to excess bond returns because due to their high frequency (i) the risk-free return is close to zero so that returns are similar to excess returns and (ii) rate changes are close to negative returns.

setting with multi-period overlapping returns we calculate standard errors using the reverse regression delta method of Wei and Wright (2013).

Table 4 reports estimates of Equation (2) for different sets of predictors. The first column displays results for level, slope, and curvature alone. Level and slope are significant, and the slope coefficient is positive, in line with previous work that found a high slope to predict falling long-term yields and high bond returns (Campbell and Shiller, 1991, Cochrane and Piazzesi, 2005).

Adding ISK roughly doubles the predictive power relative to the yields-only specification (measured by R^2) and the coefficient on ISK is highly significant. The coefficient on ISK has a negative sign, and the coefficient on the slope becomes more positive and more strongly significant. The fact that conditional yield skewness has significant predictive power even controlling for the information yields—i.e., that the predictive power of ISK is not subsumed by the shape of the yield curve—indicates a violation of the spanning hypothesis (Duffee, 2011; Bauer and Hamilton, 2018).

Estimates for individual bond returns yield similar conclusions. The predictive power of ISK for excess returns is stronger for short than for long bond maturities. We omit the results for the sake of brevity.

As a more reliable method of inference that accounts for potential small-sample problems, Bauer and Hamilton (2018) propose a bootstrap method to test the spanning hypothesis in predictive regressions for bond returns. Using their bootstrap procedure leads to a small-sample p-value on the coefficient of ISK that is below 1%, confirming the result reported in the Table. ISK exhibits only moderate autocorrelation in monthly data, with a first-order autocorrelation coefficient of 0.72, which alleviates some of the usual concerns about inference with persistent predictors.

Column (3) uses RSK instead of ISK. Realized skewness also has predictive power for bond returns, but less than implied skewness, as evidenced by the lower R^2 in column (3) than column (2). In column (4) we estimate a regression that includes both ISK and RSK. In this specification, in addition to the yield curve predictors, it is only ISK that exhibits significant predictive power, but not RSK.

Column (5) adds a different control, namelyan estimate for the trend component of nominal interest rates, or i^*, which Bauer and Rudebusch (2020) have found to be important for predicting Treasury yields and returns. Accounting for the slow-moving interest rate trend in this way further raises the predictive power for future bond returns, but ISK remains highly statistically significant.

The last column explores the relation between ISK and the survey disagreement about the 10-year yield advocated by Giacoletti, Laursen, and Singleton (2021) (GLS). The sample for this regression is shorter because the GLS variable is currently available only until November of 2018. The estimates show that ISK continues to be significant when combined with yield disagreement, and that both variables add forecasting power for future bond returns.

In additional, unreported analysis we have investigated further predictive models that control for other variables. Our earlier estimates in Section 3 documented a strong contemporaneous relationship with implied skewness for a level-slope interaction effect and for indicator variables capturing the state of the monetary policy cycle. Including these variables in the excess return regressions has no material effect on the predictive power of ISK. This supports the view that the variation in implied skewness that is *orthogonal* to the yield curve and business cycle indicators, the residual we plotted in Figure 1C, contains information that is highly relevant for the future evolution of interest rates. Lastly, the asset pricing literature has focused on the role option-implied variance (e.g., Choi, Mueller, and Vedolin, 2017). Such a measure captures the market uncertainty but does not posses directional information. Indeed, our findings are unchanged when controlling for option-implied variance or volatility using measures calculated from our Treasury options and using the TYVIX index.

Our interest rate data are the smoothed Treasury yields of Gürkaynak, Sack, and Wright (2007), but we have also run predictive regressions using the popular unsmoothed Fama-Bliss Treasury yields. Cochrane and Piazzesi (2005) famously documented in this dataset that a single linear combination of forward rates captures essentially all of the predictive power of the yield curve for future excess returns across bond maturities. Our evidence with the Fama-Bliss data, which we omit for the sake of brevity, also shows that ISK strongly and robustly predicts future bond returns, for both the averaged bond return as well as for individual returns for 2-5 years bond maturities. Importantly, this finding is robust to controlling for the usual yield factors, all five annual forward rates, or the powerful Cochrane-Piazzesi factor.

Finally, one might wonder about the role of the ZLB episode for these results. After all, this is an episode during which yield skewness was substantially elevated, and Treasury yields generally increased after the Fed lifted the policy rate off the ZLB, at least for some time. To address this issue, we estimate predictive regressions for the period before the ZLB, using a sample that ends in November 2008. Appendix A.2 shows the results, which reveal that the predictive power of conditional skewness was in fact much stronger during this first part of the sample, before the ZLB became a serious concern. In short, the ZLB episode does not contribute to the predictive power of yield skewness.

4.2 Monetary policy surprises

We now zoom in on an important source of new information for bond markets: FOMC announcements. Going back to Kuttner (2001), an extensive literature has studied the reaction of interest rates to the surprise change in short-term interest rates. Such monetary policy surprises are typically calculated based on intraday changes in money market futures rates over a tight window around the FOMC announcements (Gürkaynak, Sack, and Swanson, 2005). Several recent papers, however, have found that the high-frequency rate changes are predictable using publicly available macroeconomic data (Bauer and Swanson, 2020; Cieslak, 2018; Miranda-Agrippino, 2017).

We measure the policy "surprise" as the first principal component of intraday rate changes around the announcement that are derived from changes in Fed funds and Eurodollar futures prices, following Nakamura and Steinsson (2018). This surprise, denoted below by s_t, is a univariate summary of the shift in short- and medium-term interst rates—the change in the expected path of future policy rates, up to a term premium component. Appendix A.3 contains additional results for s_t measured as either the target surprise or the path surprise of Gürkaynak, Sack, and Swanson

(2005). We report estimates for predictive regressions

$$s_t = \beta' X_{t-1} + \varepsilon_t, \tag{3}$$

where t are days with FOMC announcements, X_{t-1} are predictors observed on the day before the announcement and ε_t is a prediction error. For statistical inference, we report White heteroskedasticity-robust standard errors, because ε_t is not serially correlated between FOMC announcements. Our sample contains 213 FOMC announcements from the beginning of 1994 (when the FOMC first started publicly stating a target for the policy rate) to June 2019 (where our data for intradaily policy surprises ends). The sample includes both scheduled and unscheduled FOMC announcements, but our results are not sensitive to the exclusion of unscheduled announcements.

Table 5 reports results showing the predictive power of skewness for interest rate changes around FOMC announcements. In most of the specifications, X_t contains the level, slope and curvature of the yield curve, based on daily Treasury yields with annual maturities from one to ten years. Subsequent specifications add ISK and RSK, which are calculated from Treasury derivative prices over the 22 days preceding the day before the FOMC announcement.[5]

Column (1) shows that yield factors alone do not have any predictive power. In Column (2) we instead use ISK in a univariate regression, which shows that by itself ISK has significant predictive power and explains close to six percent of the variance in the monetary policy surprise. Column (3) shows that adding both ISK and the yield curve completely changes the result compared to the yields-only specification in column (1): Now both the slope and ISK are highly statistically significant, and the R^2 is about 10%. The slope's coefficient is statistically negative, while the coefficient of ISK is significantly positive, mirroring the findings for the return regressions in Table 4.

If instead of ISK we include RSK as a predictor, it is also found to exhibit significant predictive power, as shown in colum (4). But in this case, the R^2 is only around

[5]ISK is taken as the mean of the daily ISK observations over these 22 days, or roughly one month, before the FOMC meeting. RSK is calculated as in Section 2 but based on sums over the 22 trading days before the FOMC meeting (instead of a calendar month). Moderate changes to these window lengths do not materially affect our results.

3%. Column (5) includes both ISK and RSK with the yield curve factors. The slope and ISK are the most significant predictors in this specification, and the predictive power is similar to the specification without RSK. Similarly to return regressions, the information in implied skewness appears more relevant for predicting future rate changes than the information in realized skewness.

Table 6 considers specifications with macroeconomic variables that have been found to predict FOMC surprises in previous studies. Our goal here is to assess whether ISK retains its predictive power even if we control for these additional predictors in our regressions. In column (1) we include the predictors considered by Cieslak (2018): the average federal funds rate over the month preceding the FOMC meeting, and annual employment growth, measured as the 12-month log-change in total nonfarm payroll employment, appropriately lagged so that it is known by the day before the FOMC announcement. In this specification, employment growth but not the Federal Funds rate exhibits predictive power. The lack of predictive power of the funds rate is partly due to our different sample period and partly due to the different policy surprise measure than in the estimates of Cieslak (2018). Using Cieslak's exact sample and regression specification we are able to replicate her results, and we still find that when we add ISK to the regression it significantly raises the predictive power.

Columns (2) to (4) add the macroeconomic variables considered by Bauer and Swanson (2020): the Brave-Butters-Kelley business cycle indicator produced by the Chicago Fed, the change in nonfarm payroll employment in the previous month (again appropriately accounting for the publication lag), and the return of the S&P 500 stock index over the three months preceding the FOMC announcement. In all three cases, both ISK and the Bauer-Swanson predictor exhibit statistically significant explanatory power for the FOMC policy surprise.

4.3 Survey forecast errors

The evidence on interest-rate predictability with skewness in Sections 4.1 and 4.2 speaks to time variation in expected excess bond returns, that is, the left-hand side of equation (1). The right-hand side of the equation tells us that the established predictability may arise because of systematic forecast errors that are related to skewness. To investigate this possibility, we use survey forecasts as proxies for subjective expectations.

Specifically, we calculate forecast errors for the 10-year Treasury yield from the Blue Chip Financial Forecasts (BCFF). This is a monthly survey that contains forecasts for the current quarter (nowcasts) and each of the next five quarters.[6] The forecast target is the quarterly average for the constant-maturity 10-year yield from the Fed's H.15 statistical release, which we obtain from FRED. We calculate errors as the difference between the realized value and the consensus forecast, which is the average of the individual forecasts.

For each forecast horizon from $h = 0$ to 5 quarters we run monthly predictive regressions of the forecast errors on information available at the time of the survey. Specifically, we estimate the regression

$$y_{q(t,h)} - \widehat{y}_t^{(h)} = \beta' X_t + \varepsilon_t^{(h)},$$

where t indexes the month of the survey forecast, $y_{q(t,h)}$ is the average ten-year yield over quarter $q(t,h)$ that contains the month $t+3h$, $\widehat{y}_t^{(h)}$ is the forecast for the average yield in quarter $q(t,h)$, X_t are predictors observable at the time the survey forecasts are made, and $\varepsilon_t^{(h)}$ is a forecast error.[7] For example, in January, February, and March, forecasts for $h = 1$ are for the average over the second quarter (April to June). We measure the predictors on the day of the BCFF survey deadline to ensure that they are observable at the time the forecast is made.[8] Our predictors X_t include ISK as well as the level, slope and curvature of the yield curve.

The forecast errors of these regressions are necessarily serially correlated due to both the monthly frequency and also overlapping observations. Because of the latter, Hansen-Hodrick standard errors are preferable to Newey-West standard errors, see

[6] The surveys conducted before 1997 extend out only four quarters.

[7] Forecasts about yields can be related to forecasts about excess returns via $E_t^s(rx_{t+1}^{(n)}) = -(n-1)E_t^s(y_{t+1}^{(n-1)}) + ny_t^{(n)} - y_t^{(1)}$.

[8] The surveys are conducted between the 23rd and 26th of the preceding month; the January survey is conducted between the 17th and the 21st of December.

Cochrane and Piazzesi (2005). We use $3(h+1)$ lags to estimate the covariance matrix of the parameter estimates.

Table 7 shows the results. For all forecast horizons, the coefficient on ISK is positive and statistically significant at the five-percent level. As before, we find that in predictive regressions with ISK and yield factors, the slope of the yield curve has a negative coefficient, which is statistically significant for horizons of three quarters and beyond.

Appendix A.4 present results for a similar analysis using the consensus forecasts in the Survey of Professional Forecasters. There we also find strong predictive power of conditional skewness for survey forecast errors at all forecast horizons.

Previous work has documented that expectation errors are particularly pronounced for short-term yields (Cieslak, 2018; Brooks, Katz, and Lustig, 2020). We have therefore also investigated the predictive power for Blue Chip forecast errors for Treasury yields of shorter maturities (1y, 2y, 5y). We found that ISK forecasts with higher accuracy the shorter the yield maturity. We omit these results for the sake of brevity.

Finally, we also investigated the role of survey forecast revisions. Coibion and Gorodnichenko (2015) found that forecast revisions are strong predictors of forecast errors for inflation and other macroeconomic variables, suggesting an important role of information rigidities for the expectation formation process. However, for long-term Treasury yields, we have not found ex-ante survey revisions to have any predictive power for ex-post forecast errors. Including revisions as additional predictors in the regressions leaves our results essentially unchanged. Notably, Coibion and Gorodnichenko (2015) suggested that information rigidities play a larger role for less persistent time series, while the ten-year Treasury yield is highly persistent, which might explain this empirical finding.

This evidence suggests that the correlation of ISK with future interest rates that we have documented above in Tables 4–6 is unlikely to be due entirely to its correlation with risk premiums. Instead, it appears that conditional yield skewness is systematically related to the difference between subjective and true expectations about future yields and bond returns, that is, to persistently biased beliefs of investors.

4.4 Skewness in the time of COVID

As we noted earlier, skewness has reversed its downward trend and reached all-time high values in the wake of the COVID pandemic. A natural question is whether the unusual circumstances have disrupted the properties of skewness established in this paper. The answer is no. In fact, the period immediately preceding the COVID lockdown and the COVID period itself serve as a nice showcase of our findings.

Figure 4 displays the main actors in the reported evidence: ISK, the ten-year Treasury yield and its survey forecasts from BCFF, and the slope of the yield curve, measured as the difference between the ten-year and three-month Treasury yields. We see that skewness was negative throughout 2019, and, in fact, sharply dropped to -1.5 in early 2020 as the pandemic was taking hold. After that, coincident with aggressive monetary and fiscal stimulus, it started climbing back and ultimately reached historically high values around 1.0 in the second half of 2020. Was skewness helpful in predicting 10-year yields during this period? Was it related to expectational errors of forecast surveys?

Early 2019 and late 2020 are two episodes where the slope was close to zero in both cases, predicting low bond returns and rising interest rates. But the level of skewness differed substantially, negative during the first and positive during the second episode. In 2019 the signal from the slope turned out to be incorrect, as yields dropped precipitously. This was correctly anticipated by the implied skewness. In 2020, while the prediction of the flat yield curve for rising long-term rates ultimately turned out to be correct, the slope was essentially unchanged over most of the year, so that it was of little use as a timely indicator of interest rate risk. Skewness, by contrast, all of a sudden rose subtantially in the middle of the year, correctly anticipating the rising long-term yields. Both of these episodes highlight the extra information in skewness that is not present in the current yield curve.

Large swings in skewness during this period indicate large expectational errors. Consistent with our regression results in Table 7, forecasters were overshooting yields in the beginning of the pandemic and then undershooting later in this episode. In fact, in late 2020 expectational errors were large as Treasury yields began a sustained ascent from historical lows (from 0.5% to 1.5%). At the time market observers were surprised by the development. Again, this was correctly predicted by skewness, which started rising in advance of the rise in yields.

Thus, the COVID episode and the sudden rise in yields seem special at a first glance. After all, yields rarely move by such large magnitudes over such short periods of time. But it wasn't so different from the vantage point of conditional skewness, which correctly anticipated both the dramatic decline in long-term Treasury yields in 2019 and early 2020, as well as their pronounced increase starting in the middle of the COVID pandemic.

5 A potential explanation of the evidence

To summarize, we have accumulated the following evidence: Treasury yields exhibit time-varying conditional skewness. A single measure of the skewness, ISK, is related to the shape of the yield curve, and predicts, after controlling for the yied curve and other usual suspects, bond excess returns and yield forecasting errors associated with professional surveys. A natural question is whether this evidence is consistent with an economic mechanism. That skewness is time-varying and is related to bond excess returns via expectation errors implies that an explanation should feature non-normal time-varying distribution and subjective beliefs.

The heterogeneous beliefs framework is an attractive candidate for capturing the evidence because, as we show below, it does not require hardwiring non-normal distribution of state variables and differences in beliefs could be interpreted as expectation errors. Heterogeneous beliefs models are known to imply time variation in risk premiums and volatility of asset returns. They also impact the spot risk-free rate. Unfortunately, there is dearth of research on the term structure of interest rates.

In this section we adopt the EGHI model to demonstrate how heterogeneous beliefs could be consistent with our evidence. We consider a special case of the framework where one of the agents knows the true distribution of inflation while the other one has an erroneous conditional expectation. This perspective is consistent with Reis

(2020) who posits that bond traders are better informed than the general public, which is proxied by surveys.

In this case, the usual disagreement between the agents becomes a measure of a bias in beliefs, computed as the difference between (biased) survey-based and (true) statistical inflation expectations, standardized by inflation's volatility. As we show in Appendix A.5, the bias in beliefs becomes a state variable, Δ_t, that affects bond yields, their departures from the normal distribution, and bond risk premiums.

In particular, bond yields are a quadratic function of the bias via the substitution effect. That makes them non-normally distributed despite the normal state variables driving the economy. Thus, the presence of the biased beliefs directly affects the distribution of yields. Finally, bias in beliefs affects time-variation in bond risk premiums. Taken together, all these features are qualitatively consistent with the presented evidence.

As a next step, we use an illustrative version of the EGHI model to understand how skewness is related to the bias. Due to the complicated dependence of yields on states, we resort to simulations to accomplish the analysis. We conduct a comparative static exercise as follows.

We simplify the model by focusing on dynamics of inflation alone. Further, we assume that Δ_t is constant. See the special case in Appendix A.5. Lastly, we simulate the true dynamics of inflation, and then compute nominal short rate and its skewness for a range of values of Δ. Figure 5 plots of the dispersion in beliefs, Δ^2, which captures the substitution effects in the interest rates, vs the corresponding value of skewness.

Yield skewness depends on the dispersion in beliefs about inflation in a non-linear fashion. For a small Δ^2, skewness is close to zero precisely because yields become close to normally distributed. Skewness turns negative for moderate size of Δ^2, reaching a minimum of -0.5 before increasing with the size of the bias. In summary, we conclude that the model is capable of generating quantitatively the skewness' scenarios that we see in the data.

The EGHI model connects biased beliefs about inflation to the distribution of yields and bond risk premiums. The presented evidence documents the connection of skewness and risk premiums to expectation error about yields. To trace out the explicit connection to the predictions of the model, we document predictability of consensus yield forecast errors based on expectation errors about inflation. Table 8 shows that in the BCFF, inflation forecast errors are significantly positively correlated with yield forecast errors. Appendix A.4 reports similar results for the SPF forecasts. These results demonstrate that expectation errors about inflation and yields are related, in line with the predictions of the model.

6 Conclusion

Our paper makes three contributions to the macro-finance literature. First, we document novel empirical patterns for the conditional skewness of Treasury yields, including a tight empirical relationship between conditional skewness and the shape of the yield curve, the business cycle, and the stance of monetary policy. Second, we show that option-based yield skewness contains useful forward-looking information for interest rates, including predictive power for survey forecast errors. The evidence suggests that conditional skewness captures biased beliefs about future interest rates. Third, we argue that our empirical findings can be rationalized by a simple theoretical framework with heterogeneous beliefs.

References

Ahn, Dong-Hyun, Robert F. Dittmar, and A. Ronald Gallant, 2002, Quadratic Term Structure Models: Theory and Evidence, *Review of Financial Studies* 15, 243–288.

Ang, Andrew, and Monika Piazzesi, 2003, A No-Arbitrage Vector Regression of Term Structure Dynamics with Macroeconomic and Latent Variables, *Journal of Monetary Economics* 50, 745–787.

Bacchetta, Philippe, Elmar Mertens, and Eric van Wincoop, 2009, Predictability in financial markets: What do survey expectations tell us?, *Journal of International Money and Finance* 28, 406–426.

Bakshi, Gurdip, and Dilip Madan, 2000, Spanning and derivative-security valuation, *Journal of Financial Economics* 55, 205–238.

Balduzzi, Pierluigi, Giuseppe Bertola, and Silverio Foresi, 1997, A model of target changes and the term structure of interest rates, *Journal of Monetary Economics* 39, 223–249.

Balduzzi, Pierluigi, Edwin J. Elton, and T. Clifton Green, 2001, Economic news and bond prices: Evidence from the US Treasury market, *Journal of Financial and Quantitative analysis* 36, 523–543.

Basak, Suleyman, 2005, Asset pricing with heterogenous beleifs, *Journal of Banking and Finance* 29, 2849–2881.

Bauer, Michael D., and James D. Hamilton, 2018, Robust Bond Risk Premia, *Review of Financial Studies* 31, 399–448.

Bauer, Michael D., and Glenn D. Rudebusch, 2020, Interest Rates Under Falling Stars, *American Economic Review* 110, 1316–1354.

Bauer, Michael D., and Eric T. Swanson, 2020, The Fed's Response to Economic News Explains the "Fed Information Effect", Working Paper 2020-06 Federal Reserve Bank of San Francisco.

Beber, Alessandro, and Michael W. Brandt, 2006, The Effect of Macroeconomic News on Beliefs and Preferences: Evidence from the Options Market, *Journal of Monetary Economics* 53, 1997–2039.

Bikbov, Ruslan, and Mikhail Chernov, 2009, Unspanned stochastic volatility in affine models: evidence from eurodollar futures and options, *Management Science* 55, 1292–1305.

Brooks, Jordan, Michael Katz, and Hanno Lustig, 2020, Post-FOMC Announcement Drift in U.S. Bond Markets, Working Paper.

Buraschi, Andrea, Ilaria Piatti, and Paul Whelan, 2019, Subjective Bond Risk Premia and Belief Aggregation, *Saïd Business School WP* 6, 2016–36.

Campbell, John Y., and Robert J. Shiller, 1991, Yield Spreads and Interest Rate Movements: A Bird's Eye View, *Review of Economic Studies* 58, 495–514.

Chernov, Mikhail, Jeremy Graveline, and Irina Zviadadze, 2018, Crash Risk in Currency Returns, *Journal of Fnancial and Quantitative Analysis* 53, 137–170.

Choi, Hoyong, Philippe Mueller, and Andrea Vedolin, 2017, Bond Variance Risk Premiums, *Review of Finance* 21, 987–1022.

Cieslak, Anna, 2018, Short-Rate Expectations and Unexpected Returns in Treasury Bonds, *The Review of Financial Studies* 31, 3265–3306.

Cieslak, Anna, and Pavol Povala, 2015, Expected Returns in Treasury Bonds, *Review of Financial Studies* 28, 2859–2901.

Cieslak, Anna, and Annette Vissing-Jorgensen, 2020, The Economics of the Fed Put, *The Review of Financial Studies* hhaa116.

Cochrane, John H., and Monika Piazzesi, 2005, Bond Risk Premia, *American Economic Review* 95, 138–160.

Coibion, Olivier, and Yuriy Gorodnichenko, 2015, Information rigidity and the expectations formation process: A simple framework and new facts, *American Economic Review* 105, 2644–2678.

Das, Sanjiv, 2002, The surprise element: jumps in interest rates, *Journal of Econometrics* 106, 27–65.

Detemple, Jerome, and Shashidhar Murthy, 1994, Intertemporal asset pricing with heterogenous beleifs, *Journal of Economic Theory* 62, 294–320.

Duffee, Gregory R., 2011, Information In (and Not In) the Term Structure, *Review of Financial Studies* 24, 2895–2934.

Ehling, Paul, Michael Gallmeyer, Christian Heyerdahl-Larsen, and Philipp Illeditsch, 2018, Disagreement about inflation and the yield curve, *Journal of Financial Economics* 127, 459–484.

Eriksson, Anders, Eric Ghysels, and Fangfang Wang, 2009, The normal inverse Gaussian distribution and the pricing of derivatives, *Journal of Derivatives* 16, 23–37.

Fleming, Michael, and Eli Remolona, 2001, The Term Structure of Announcement Effects, BIS Working Paper No. 71.

Giacoletti, Marco, Kristoffer Laursen, and Kenneth Singleton, 2021, Learning From Disagreement in the U.S. Treasury Bond Market, *Journal of Finance* 76, 395–441.

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007, The U.S. Treasury Yield Curve: 1961 to the Present, *Journal of Monetary Economics* 54, 2291–2304.

Gürkaynak, Refet S., Brian P. Sack, and Eric T. Swanson, 2005, Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements, *International Journal of Central Banking* 1, 55–93.

Harrison, J. Michael, and David Kreps, 1978, Speculative investor behavior in a stock market with heterogenous expectations, *Quarterly Journal of Economics* 92, 323–336.

Hattori, Masazumi, Andreas Schrimpf, and Vladyslav Sushko, 2016, The Response of Tail Risk Perceptions to Unconventional Monetary Policy, *American Economic Journal: Macroeconomics* 8, 111–136.

Hodrick, Robert J., 1992, Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement, *Review of Financial Studies* 5, 357–386.

Jiang, George, and Yisong Tian, 2005, Model-Free Implied Volatility and Its Information Content, *Review of Financial Studies* 18, 1305–1342.

Johannes, Michael, 2004, The statistical and economic role of jumps in continuous-time interest rate models, *Journal of Finance* 59, 227–260.

Kuttner, Kenneth N., 2001, Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market, *Journal of Monetary Economics* 47, 523–544.

Li, Xinyang, 2020, Bond Implied Risks Around Macroeconomic Announcements, Working Paper.

Ludvigson, Sydney, and Serena Ng, 2009, Macro Factors in Bond Risk Premia, *Review of Financial Studies* 22, 5027–5067.

Mertens, Thomas, and John Williams, 2020, What to Expect from the Lower Bound on Interest Rates: Evidence from Derivatives Prices, American Economic Review, forthcoming.

Miranda-Agrippino, Silvia, 2017, Unsurprising Shocks: Information, Premia, and the Monetary Transmission, Staff Working Paper 626 Bank of England.

Nakamura, Emi, and Jón Steinsson, 2018, High-Frequency Identification of Monetary Non-Neutrality: The Information Effect, *Quarterly Journal of Economics* 133, 1283–1330.

Neuberger, Anthony, 2012, Realized Skewness, *The Review of Financial Studies* 25, 3423–3455.

Piazzesi, Monika, 2001, An econometric model of the yield curve with macroeconomic jump effects, *NBER working paper 8246*.

Piazzesi, Monika, 2005, Bond Yields and the Federal Reserve, *Journal of Political Economy* 113, 311–344.

Piazzesi, Monika, Juliana Salomao, and Martin Schneider, 2015, Trend and Cycle in Bond Premia, unpublished manuscript.

Reis, Ricardo, 2020, The People versus the Markets: A Parsimonious Model of Inflation Expectations, Working Paper.

Rudebusch, Glenn D., and Tao Wu, 2008, A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy, *Economic Journal* 118, 906–926.

Schmeling, Maik, Andreas Schrimpf, and Sigurd Steffensen, 2020, Monetary Policy Expectation Errors, Working Paper.

Trolle, Anders B., and Eduardo S. Schwartz, 2014, The Swaption Cube, *Review of Financial Studies* 27, 2307–2353.

Wei, Min, and Jonathan H Wright, 2013, Reverse Regressions And Long-Horizon Forecasting, *Journal of Applied Econometrics* 28, 353–371.

Xiong, Wei, and Hongjun Yan, 2010, Heterogenous expectations and bond markets, *Review of Financial Studies* 23, 1433–1466.

A Appendix

A.1 Calculating option-implied moments of Treasury yields

Our Treasury derivatives data are end-of-day prices of Treasury futures and options from CME.[9] We focus on the 10-year T-note futures contract (or "TY"). The deliverable securities for the TY contract are "U.S. Treasury notes with a remaining term to maturity of at least six and a half years, but not more than 10 years" (according to the CME contract specifications). The contract expirations are at the end of each calendar quarter, and at each point in time three consecutive quarterly contracts are available; the exact delivery date is roughly in the third week of the month. The first quarterly contract is the most active, until about 2-3 weeks before expiration when trading in the subsequent quarterly contract becomes more active. Therefore, when working with futures prices (e.g., for calculating sample moments or realized moments of price changes), we always use the first quarterly expiration that is not in the current calendar month (e.g., we use the March contract until the end of February, and the June contract starting in the beginning of March).

The options on the TY contract are available for three quarterly and three serial (monthly) expirations, and they each exercise into the next futures contract. For example, February and March options exercise into the March futures contract, and April options exercise into June futures contract. The last trading day for each options contract is the "2nd last business day of the month prior to the contract month" so that trading for the March options ends at the end of February. We denote by t the current trading day and by T the last trading day (or expiration date) of an options contract. For most of our analysis we focus on the first quarterly option expiration. In some cases we linearly interpolate option-implied moments to a constant horizon, and then we use 0.2 years as the horizon which is about the average maturity of all option contracts (across all expirations, strikes and put/call prices), and interpolate based on the data for the two expirations surrounding this horizon.

Based on option prices on day t for the contract expiration T we can calculate conditional market-based/risk-neutral moments for the price of the underlying futures contract at the time of the option expiration, F_T. The implied risk-neutral variance is

$$\begin{aligned} Var_t F_T = E_t(F_T - F_t)^2 &= 2\left[\int_{F_t}^{\infty} C(K)dK + \int_0^{F_t} P(K)dK\right] \\ &= 2\int_0^{\infty} C(K) - \max(0, F_T - K)dK \end{aligned}$$

where all moments are under the time-T forward measure, we treat F_t as the forward price for simplicity, and the forward call and put prices for options with strike K are $C(K)$ and $P(K)$. Because expectations are under the T-forward measure, $E_t F_T = F_t$, $C(K) = E_t \max(0, F_T - K)$ and $P(K) = E_t \max(0, K - F_T)$. The second line follows from put call parity, $C(K) - P(K) = F_T - K$.

[9] For details see https://www.cmegroup.com/trading/interest-rates/us-treasury.html.

The implied third moment is

$$E_t(F_T - F_t)^3 = 6\left[\int_{F_t}^{\infty}(K - F_t)C(K)dK - \int_0^{F_t}(F_t - K)P(K)dK\right]$$
$$= 6\int_0^{\infty}(K - F_t)(C(K) - \max(0, F_T - K))dK.$$

See also Trolle and Schwartz (2014) who use similar formulas for calculating swaption-implied moments for future swap yields. The implied skewness coefficient is

$$skew_{t,T}^F = \frac{E_t(F_T - F_t)^3}{(Var_t F_T)^{3/2}}$$

We now describe how we implement these measures empirically. In what follows σ is the normal implied volatility (IV) for at-the-money options. Normal IV, the most common way to measure IV in bond markets, is based on the Bachelier model and measures the volatility of future price changes under the assumption that they are Gaussian. First, we filter our options data to reduce the impact of measurement error and eliminate data errors, similar to Beber and Brandt (2006). Specifically, we exclude options that

- have maturity of at most two weeks
- have prices of at most two ticks (2/64)
- have relative moneyness greater than 15, i.e., $(F_t - K)/\sqrt{(T-t)\sigma^2}$ is at most 15 in absolute value (options that are further out of the money tend to have unreliable/implausible IVs),
- are too far out of the money, with absolute moneyness of less than -15 (the absolute moneyness is $F - K$ for calls and $K - F$ for puts),
- have distinct duplicate prices for the same strike (using the IVs and other prices we can eliminate the erroneous price by hand),
- have prices which are not monotone across strikes, or
- violate the no-arbitrage condition that the price is no lower than the intrinsic value.

Then we calculate implied moments for each pair (t, T) if we observe at least five option prices (puts and calls across all strikes) in the following way:

1. We select all option prices that are ATM/OTM
2. We calculate the normal IVs for these observed prices.
3. We fit a curve in strike-IV space by linearly interpolating IVs and, outside the range of observed prices, using IVs at the endpoints of the range.
4. We obtain a continuous price function $C(X)$ by mapping the IVs back to call prices using the Bachelier pricing formula.
5. We approximate the required integrals using trapezoid rule for grid of strike prices from $F_t - 10$ to $F_t + 10$ with 200 grid points (see also Jiang and Tian, 2005).

As a result we have, for each trading day t and option expiration T, conditional model-implied variances and skewness coefficients for the change in the futures price between t and T.

With the moments for futures prices in hand we can also calculate certain moments for changes in the yields of the cheapest-to-deliver (CTD) bond. The reason is that for small changes, the relationship between changes in futures prices and changes in the CTD yield is approximately linear. The "dollar value of a basis point" (DV01) is the negative sensitivity of the futures price (in points) to a change in the CTD yield (in basis points). Denoting the change in the futures price as ΔF and the change in the CTD yield by Δy, we have

$$\Delta y \approx -\frac{\Delta F}{DV01}.$$

Under the assumption that the change in the CTD yield until expiration, $y_T - y_t$, is small, and that $DV01$ remains approximatley unchanged between t and T, we can obtain risk-neutral moments for future yields as

$$Var_t y_T \approx \frac{Var_t F_T}{(DV01)^2}, \quad E_t(y_T - y_t)^3 \approx -\frac{E_t(F_T - F_t)^3}{(DV01)^3}, \quad skew^y_{t,T} \approx -skew^F_{t,T}$$

The $DV01$ data, as well as any information about the CTD bonds, becomes available on Bloomberg in 2004. But this information is not required for the skewness coefficient, since skewness of yield changes is approximately equal simply to the negative of the skewness of futures price changes.

Our derivation and implementation abstracts from the fact that Treasury options are American options on futures contracts, and not, as assumed, European options on forward contracts. Existing results suggest that accounting for early exercise would lead to only minor adjustments; see Bikbov and Chernov (2009) and Choi, Mueller, and Vedolin (2017). In addition, since we only use out-of-the-money options any adjustment for early exercise would be minimal, since there are no dividends and the early-exercise premium increases with the moneyness of options.

A.2 Additional results for Treasury bond returns

Table A1 shows predictive regressions for excess bond returns similar to those in Table 4, but for a sample that ends in November 2008, before the Fed lowered the policy rate to the ZLB. Limiting the sample period in this way substantially increases the coefficient on ISK, and with only the exception of the univariate regression in column 2 it also raises the statistical significance of this coefficient and the R^2 of the regression. Evidently, conditional yield skewness had even more predictive power for bond returns before the ZLB period.

A.3 Additional results for FOMC announcement surprises

Since Gürkaynak, Sack, and Swanson (2005) the literature on high-frequency event studies of FOMC announcements has focused on two measures of the policy surprises: a target surprise which, similar to the original measure proposed by Kuttner (2001), measures the surprise change in the federal funds rate, and a path surprise which captures the change in the expected policy path that is orthogonal to the target surprise. The two surprises are the first two principal components of the high-frequency changes in different money market futures rates, appropriately rotated and scaled (for details see Gürkaynak, Sack, and Swanson, 2005).

Table A2 shows estimates of predictive regressions for the target surprise (columns 1-5) and the path surprise (6-10). Generally, ISK contains predictive power for both components of the policy surprise. However, in regressions with macro variables, the predictive power tends to be stronger for the path surprise. In additional, unreported results we have found that excluding the 9 unscheduled announcements in our dataset lowers the predictability of the target surprise but raises the predictability of the path surprise.

A.4 Additional results for SPF forecast errors

Here we present additional evidence using the quarterly Survey of Professional Forecasters (SPF). As in the BCFF, the forecast target is the quarterly average for the constant-maturity 10-year yield from the Fed's H.15 statistical release. Forecasts are reported for the current quarter (nowcasts) and each of the subsequent four quarters. As the SPF consensus forecast we take the median of the individual forecasts.

We run predictive regressions of the form

$$y_{t+h} - \widehat{y}_t^{(h)} = \beta' X_t + \varepsilon_{t,t+h}, \tag{A.1}$$

where t indexes the quarterly SPF surveys, y_t is the average 10-year yield in quarter t, $\widehat{y}_t^{(h)}$ is the survey consensus forecast made in quarter t for the average 10-year yield in quarter $t+h$, h ranges from 0 to 4, X_t is a vector with predictors, and $\varepsilon_{t,t+h}$ is a forecast error. To ensure that the predictors X_t are observable at the time the forecast is made, we take observations on the day before the response deadline of the survey. Because the forecast errors $\varepsilon_{t,t+h}$ are serially correlated we use Hansen-Hodrick standard errors with h lags.

Table A3 shows the results. For each forecast horizon, we estimate two specifications, one with ISK only, and one that also includes yield factors. We find that ISK has statistically significant predictive power for all forecast horizons. The specifications that also include yield curve factors show that the slope tends to have additional predictive power for $h > 0$. As before, we find that the slope has a negative coefficient while ISK has a positive coefficient.

Paralleling our analysis for the Blue Chip surveys (see Table 8 and the discussion in Section 5, we also consider the relationship between yield forecast errors and inflation forecast errors in the SPF surveys. The results in Table A4 show that there is generally a positive correlation, with a relationship that is strongly statistically significant for the one-quarter-ahead forecast, and marginally significant for the two- and four-quarter horizons. Overall these results are consistent with our findings from the Blue Chip surveys.

A.5 A model of biased beliefs

The model is adopted from Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018).

Setup

Consumption growth follows

$$dC_t/C_t = \mu_C dt + \sigma_C dz_{C,t}.$$

Inflation follows:

$$\begin{aligned} d\Pi_t/\Pi_t &= x_t dt + \sigma_\Pi dz_{\Pi,t}, \\ dx_t &= \kappa(\bar{x} - x_t)dt + \sigma_x dz_{x,t}. \end{aligned}$$

Expected inflaton x_t is not observable. There are two agents in the economy who disagree about its true dynamics, specifically its volatility σ_x. We assume that agent 1 is correct, that is $\sigma_x^1 = \sigma_x$. Agent 2 represents the consensus survey forecast, $\sigma_x^2 = \sigma_x^s$.

As a result, the second agent's perceived inflation and its expectation follow:

$$\begin{aligned} d\Pi_t/\Pi_t &= x_t^s dt + \sigma_\Pi dz_{\Pi,t}^s, \\ dx_t^s &= \kappa(\bar{x} - x_t^s)dt + \widehat{\sigma}_x^s dz_{\Pi,t}^s, \\ dz_{\Pi,t}^s &= dz_{\Pi,t} + \sigma_\Pi^{-1}(x_t - x_t^s)dt, \\ \widehat{\sigma}_x^s &= \sigma_\Pi \left(\sqrt{\kappa^2 + (\sigma_\Pi^{-1}\sigma_x^s)^2} - \kappa \right). \end{aligned}$$

The first agent does not observe x_t either, thus its filtered dynamics follow:

$$\begin{aligned} dx_t &= \kappa(\bar{x} - x_t)dt + \widehat{\sigma}_x dz_{\Pi,t}, \\ \widehat{\sigma}_x &= \sigma_\Pi \left(\sqrt{\kappa^2 + (\sigma_\Pi^{-1}\sigma_x)^2} - \kappa \right). \end{aligned}$$

The expectation error is defined as

$$dz_{\Pi,t} - dz_{\Pi,t}^s \equiv \Delta_t dt.$$

We can derive dynamics of the bias in beliefs Δ_t as

$$d\Delta_t = -\beta \Delta_t dt + \sigma_\Delta dz_{\Pi,t}, \quad \beta = \kappa + \sigma_\Pi^{-1}\widehat{\sigma}_x^s, \quad \sigma_\Delta = \sigma_\Pi^{-1}(\widehat{\sigma}_x^s - \widehat{\sigma}_x).$$

Let $\mathcal{P}$ and $\mathcal{P}^s$ denote the true and subjective probabilities, respectively. Let ξ_t and ξ_t^s denote the state-price density (SPD) under the probability $\mathcal{P}$ and $\mathcal{P}^s$, respectively. E^s is expectation taken under $\mathcal{P}^s$. Agents 1 and 2 solve their consumption-savings problems given by, respectively,

$$\begin{aligned} &\max E \left(\int_0^T e^{-\rho t} u(C_t^1/H_t) dt \right) \text{ s.t. } E \left(\int_0^T \xi_t C_t^1 dt \right) \leq w_0^1, \\ &\max E^s \left(\int_0^T e^{-\rho t} u(C_t^2/H_t) dt \right) \text{ s.t. } E^s \left(\int_0^T \xi_t^s C_t^2 dt \right) \leq w_0^2, \end{aligned}$$

where

$$u(X) \quad \equiv \quad X^{1-\gamma}/(1-\gamma),$$

and habit H_t is

$$\log H_t \quad \equiv \quad \log H_0 \cdot e^{-\delta t} + \delta \int_0^t e^{-\delta(t-u)} \log C_u du.$$

Dynamics of C_t and H_t imply that the relative log output $\omega_t = \log(C_t/H_t)$ follows

$$d\omega_t = \delta(\bar{\omega} - \omega_t)dt + \sigma_C dz_{C,t}, \quad \bar{\omega} = (\mu_C - \sigma_C^2/2)/\delta.$$

Results

Consumption allocations and state price densities. Denote the likelihood ratio by $\lambda_t = d\mathcal{P}/d\mathcal{P}^s = y^{-1}\xi_t/\xi_t^s$, where $y = y^2/y^1$, and y^i is the constant Lagrange multiplier from the respective budget constraint. Optimal consumption allocations are

$$C_t^1 = f(\lambda_t)C_t, \quad C_t^2 = (1 - f(\lambda_t))C_t, \quad f(\lambda_t) = (1 + (y\lambda_t)^{1/\gamma})^{-1}.$$

The state price densities are:

$$\begin{aligned} \xi_t &= (y^1)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} f(\lambda_t)^{-\gamma} \\ &= (y^1)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} (1 + (y\lambda_t)^{1/\gamma})^{\gamma} \\ &= \sum_{k=0}^{\gamma} \binom{\gamma}{k} (y^1)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} (y\lambda_t)^{k/\gamma}, \\ \xi_t^s &= (y^2)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} (1 - f(\lambda_t))^{-\gamma}. \end{aligned}$$

Note that y^i and y cancel out in the SDF, ξ_T^i/ξ_t^i.

Lastly,

$$d\lambda_t = \Delta_t \lambda_t dz_{\Pi,t}.$$

Bond pricing. Set $y = 1$ w.l.o.g. The real bond price is, for integer γ,

$$\begin{aligned} B_{t,T} &= E_t(\xi_T/\xi_t) = \sum_{k=0}^{\gamma} w_t^{(k)} E_t \left[e^{-\rho(T-t)} \left(\frac{C_T}{C_t}\right)^{-\gamma} \left(\frac{H_T}{H_t}\right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t}\right)^{k/\gamma} \right] = \sum_{k=0}^{\gamma} w_t^{(k)} E_t \left[\frac{\xi_{C,T}^{(k)}}{\xi_{C,t}^{(k)}} \right], \\ w_t^{(k)} &= \binom{\gamma}{k} \lambda_t^{k/\gamma} (1 + \lambda_t^{1/\gamma})^{-\gamma}, \\ \xi_{C,t}^{(k)} &= e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} \lambda_t^{k/\lambda}. \end{aligned}$$

Then

$$\begin{aligned} d\xi_{C,t}^{(k)}/\xi_{C,t}^{(k)} &= -r_t^{(k)}dt - \gamma\sigma_C \cdot dz_{C,t} - \gamma^{-1}k\Delta_t \cdot dz_{\Pi,t}, \\ r_t^{(k)} &= \rho + \gamma\mu_C - \gamma(\gamma+1)\sigma_C^2/2 - \delta(\gamma-1)\omega_t + \frac{1}{2}\frac{k}{\gamma}\left(1 - \frac{k}{\gamma}\right)\Delta_t^2. \end{aligned}$$

These expressions imply exponentially quadratic form in the states Δ_t and ω_t. Thus, bond prices are weighted averages of exponentially quadratic functions of Gaussian state variables. The weights $w_t^{(k)}$, which add up to 1, are affected by the bias in beliefs via λ_t.

The real short rate is obtained by applying Ito's lemma to ξ_t and picking out the drift of the result:

$$r_t = \rho + \gamma(\mu_C - \sigma_C^2/2) - \gamma^2\sigma_C^2/2 - \delta(\gamma-1)\omega_t + \frac{1}{2}(1 - IES)\, f(\lambda_t)(1 - f(\lambda_t))\Delta_t^2,$$

where $IES = \gamma^{-1}$ stands for intertemporal elasticity of substitution. $IES < 1$ for a risk averse agent. The terms in the expression for r_t are rate of time preference, consumption smoothing over time, precautionary savings, habit, and substitution effect.

The nominal bond price is

$$\begin{aligned} P_{t,T} &= E_t(\xi_T/\xi_t \cdot \Pi_t/\Pi_T) = \sum_{k=0}^{\gamma} w_t^{(k)} E_t\left[e^{-\rho(T-t)}\left(\frac{C_T}{C_t}\right)^{-\gamma}\left(\frac{H_T}{H_t}\right)^{\gamma-1}\left(\frac{\lambda_T}{\lambda_t}\right)^{k/\gamma}\frac{\Pi_t}{\Pi_T}\right] \\ &= \sum_{k=0}^{\gamma} w_t^{(k)} E_t\left[\frac{\xi_{\Pi,T}^{(k)}}{\xi_{\Pi,t}^{(k)}}\right], \end{aligned}$$

where $\xi_{\Pi,t}^{(k)} = e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}\lambda_t^{k/\lambda}\Pi_t^{-1}$. Then

$$\begin{aligned} d\xi_{\Pi,t}^{(k)}/\xi_{\Pi,t}^{(k)} &= -i_t^{(k)} - \gamma\sigma_C \cdot dz_{C,t} - (\sigma_\Pi + \gamma^{-1}k\Delta_t)\cdot dz_{\Pi,t}, \\ i_t^{(k)} &= \rho + \gamma\mu_C - \gamma(\gamma+1)\sigma_C^2/2 - \sigma_\Pi^2 - \delta(\gamma-1)\omega_t + x_t + \frac{k}{\gamma}\Delta_t + \frac{1}{2}\frac{k}{\gamma}\left(1 - \frac{k}{\gamma}\right)\Delta_t^2. \end{aligned}$$

These expressions imply exponentially quadratic form in states x_t, Δ_t, and ω_t. Thus, bond yields, $-(T-t)^{-1}\log P_{t,T}$ are non-normal because they are complicated non-linear functions of Gaussian state variables.

The nominal short rate is

$$i_t = r_t + \sigma_\Pi(1 - f(\lambda_t))\Delta_t + x_t - \sigma_\Pi^2.$$

When there is no bias in beliefs, $\Delta_t = 0$, nominal bond yields become linear functions of the Gaussian state variables x_t and ω_t. Thus, Δ_t controls non-normality of yields, and their skewness, in particular.

Bond risk premiums. If $\Delta_t = 0$, the bond risk premiums are constant as implied by the expression for the SPD $\xi_{\Pi,t}^{(k)}$. It is also straightforward if cumbersome to show explicit bond risk premium dependence on Δ_t.

Write the nominal bond price as

$$P_{t,T}=\sum_{k=0}^{\gamma}w_t^{(k)}P_{t,T}^{(k)},$$

where $P_{t,T}^{(k)}$ are the artificial (exponential quadratic) bond prices corresponding to the SPD $\xi_{\Pi,t}^{(k)}$.

Then,

$$\begin{aligned}E_t\left(\frac{dP_{t,T}}{P_{t,T}}\right) &= \sum_{k=0}^{\gamma}\left[\frac{P_{t,T}^{(k)}}{P_{t,T}}E_t\left(dw_t^{(k)}\right)+w_t^{(k)}E_t\left(\frac{dP_{t,T}^{(k)}}{P_{t,T}}\right)\right] \\ &= \sum_{k=0}^{\gamma}w_t^{(k)}\frac{P_{t,T}^{(k)}}{P_{t,T}}\cdot E_t\left(\frac{dP_{t,T}^{(k)}}{P_{t,T}^{(k)}}\right).\end{aligned}$$

Expected bond return in each artificial economy is going to be the corresponding risk-free rate, $i_t^{(k)}$, plus a linear function of the prices of risk, one of which is constant, $\gamma\sigma_c$, and the other one is linear in disagreement, $\sigma_\Pi+\gamma^{-1}k\Delta_t$. Continuing the previous expression, one can then write:

$$\frac{1}{dt}E_t\left(\frac{dP_{t,T}}{P_{t,T}}\right) = \sum_{k=0}^{\gamma}\frac{w_t^{(k)}P_{t,T}^{(k)}}{P_{t,T}}\cdot\left(i_t^{(k)}+\alpha_t^{(k)}+\beta_t^{(k)}\gamma\sigma_c+\gamma_t^{(k)}(\sigma_\Pi+\gamma^{-1}k\Delta_t)\right),$$

where α, β and γ reflect sensitivities of a bond price w.r.t. to its factors (see, e.g., Ahn, Dittmar, and Gallant, 2002 for explicit expressions). Thus, the risk premium is:

$$\frac{1}{dt}E_t\left(\frac{dP_{t,T}}{P_{t,T}}-i_t dt\right)=\sum_{k=0}^{\gamma}\frac{w_t^{(k)}P_{t,T}^{(k)}}{P_{t,T}}\times$$
$$\left[\widetilde{\alpha}_t^{(k)}+\left(\frac{k}{\gamma}-\sigma_\Pi(1-f(\lambda_t))+\gamma_t^{(k)}\gamma^{-1}k\right)\Delta_t+\left(\frac{k}{\gamma}\left(1-\frac{k}{\gamma}\right)-\left(1-\frac{1}{\gamma}\right)f(\lambda_t)(1-f(\lambda_t))\right)\frac{\Delta_t^2}{2}\right].$$

Special case: static bias

This model is referred to as GBM in EGHI. In this case $x_t=x$, $x_t^s=x^s$, and $C_t=H_t=1$. Then

$$\begin{aligned}dz_{\Pi,t}^s &= dz_{\Pi,t}+\sigma_\Pi^{-1}(x-x^s)dt, \\ dz_{\Pi,t}-dz_{\Pi,t}^s &= \sigma_\Pi^{-1}(x^s-x)dt\equiv\Delta dt, \\ d\lambda_t &= \lambda_t\Delta dz_{\Pi,t}.\end{aligned}$$

Bond pricing. The real bond price is exactly the same, but λ_t has a different dynamic. That will show up in the set of artificial SPDs $\xi_{C,t}^{(k)}$:

$$\begin{aligned}d\xi_{C,t}^{(k)}/\xi_{C,t}^{(k)} &= -r_t^{(k)}-\gamma^{-1}k\Delta\cdot dz_{\Pi,t}, \\ r_t^{(k)} &= \rho+\frac{1}{2}\frac{k}{\gamma}\left(1-\frac{k}{\gamma}\right)\Delta^2.\end{aligned}$$

As a result, the real short rate is

$$r_t = \rho + \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)\Delta^2 f(\lambda_t)(1 - f(\lambda_t)).$$

The nominal yields follow by analogy:

$$i_t = r_t + \sigma_\Pi(1 - f(\lambda_t))\Delta + x - \sigma_\Pi^2.$$

Figure 1: **Yield skewness.** Panel (A) displays monthly realized Treasury yield skewness, calculated from changes in daily Treasury futures prices and implied volatilities, with a 12-month moving average (blue line). Panel (B) plot daily implied Treasury yield skewness, calculated from options on Treasury futures and interpolated to a constant horizon of 0.2 years, with a 250-day moving average (blue line). Panel (C) shows residual skewness from a regression of monthly implied skewness on yield-curve factors (specification 4 in Table 3), with a 12-month moving average (blue line). Sample period: January 2, 1990, to May 28, 2021.

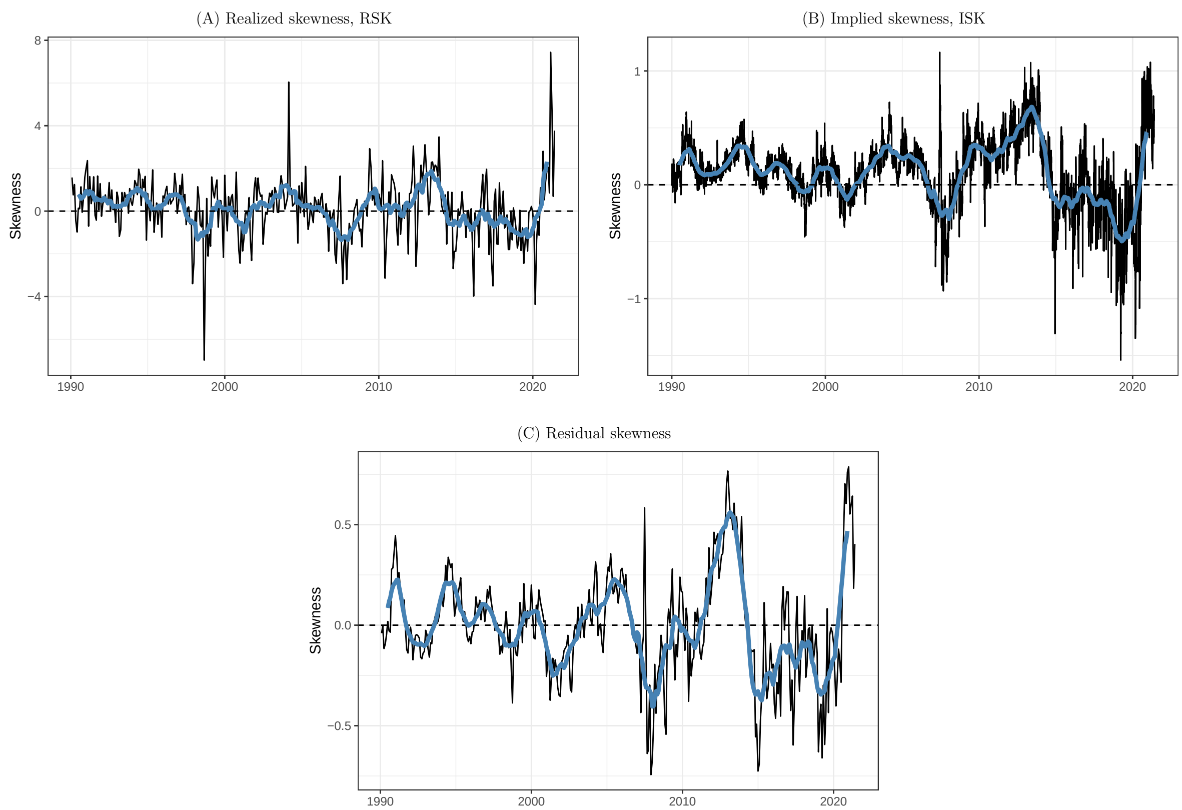

Figure 2: **Skewness and interest rates.** Option-implied yield skewness (left axis) with two-year and 10-year Treasury yields (right axis). Annual moving averages of daily values. Turquoise/orange shaded areas indicate monetary policy easing/tightening cycles (based on changes in the fed funds rate). Sample period: January 1990 to May 2021.

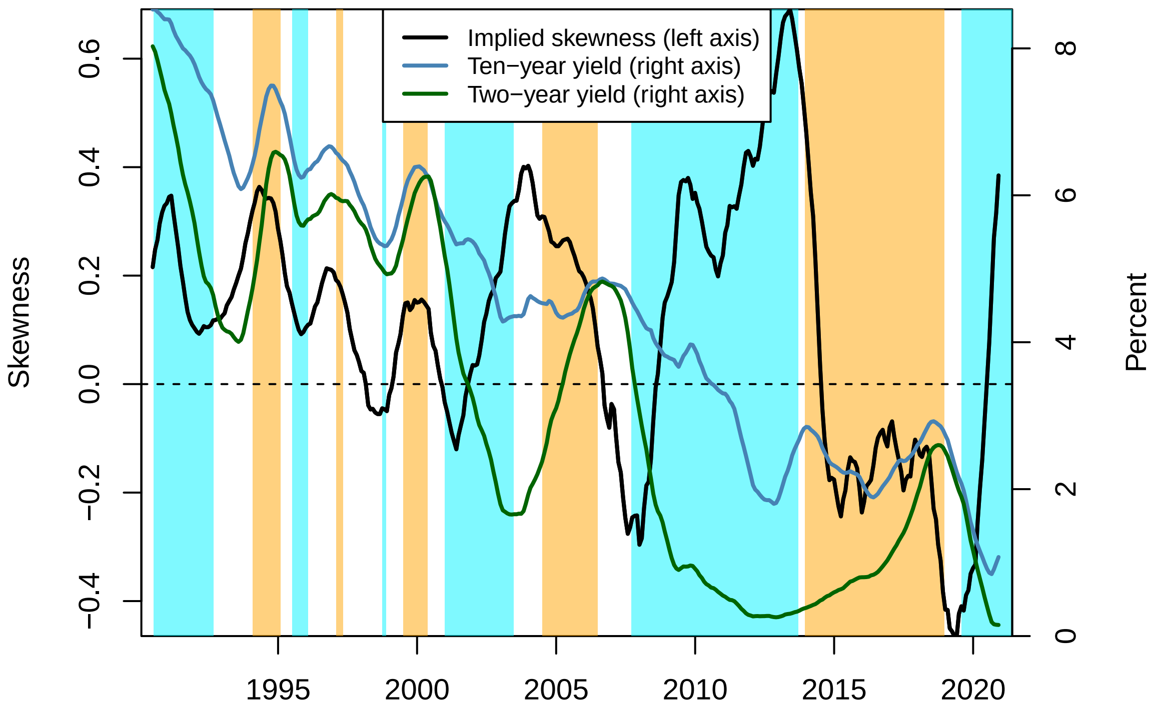

Figure 3: **Densities for future yields at ZLB.** Option-implied probabilities densities for future CTD bond yield given market prices on December 31, 2013 (left panel) and June 16, 2020 (right panel). Red shaded area indicates 1st percentile.

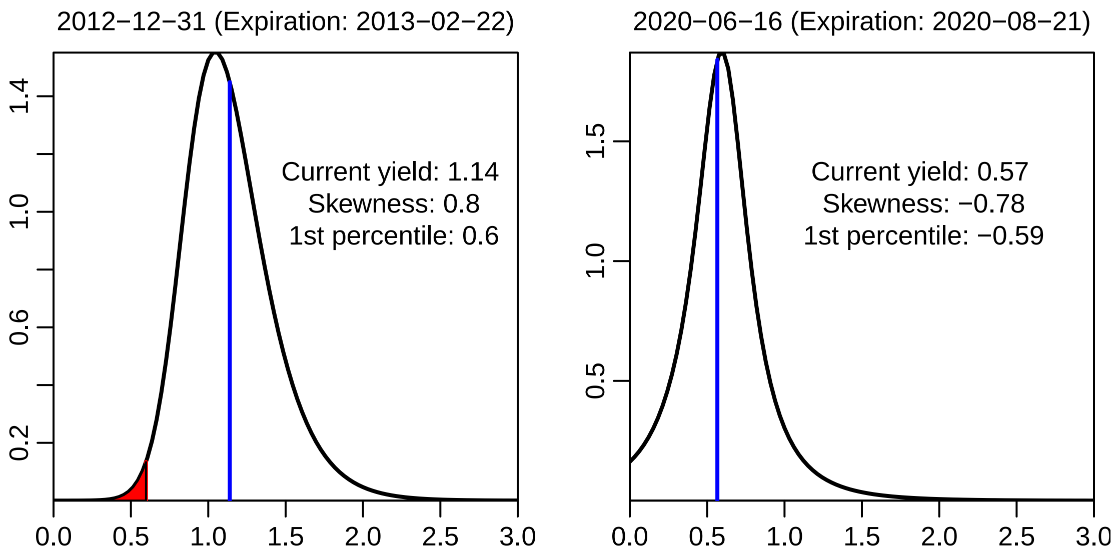

Figure 4: **Skewness and interest rates since 2019.** Ten-year Treasury yield, yield forecasts from Blue Chip Financial Forecasts, option-implied yield skewness, and slope of the yield curve (measured as difference between ten-year and three-month yield). Sample period: January 2, 2019, to May 28, 2021.

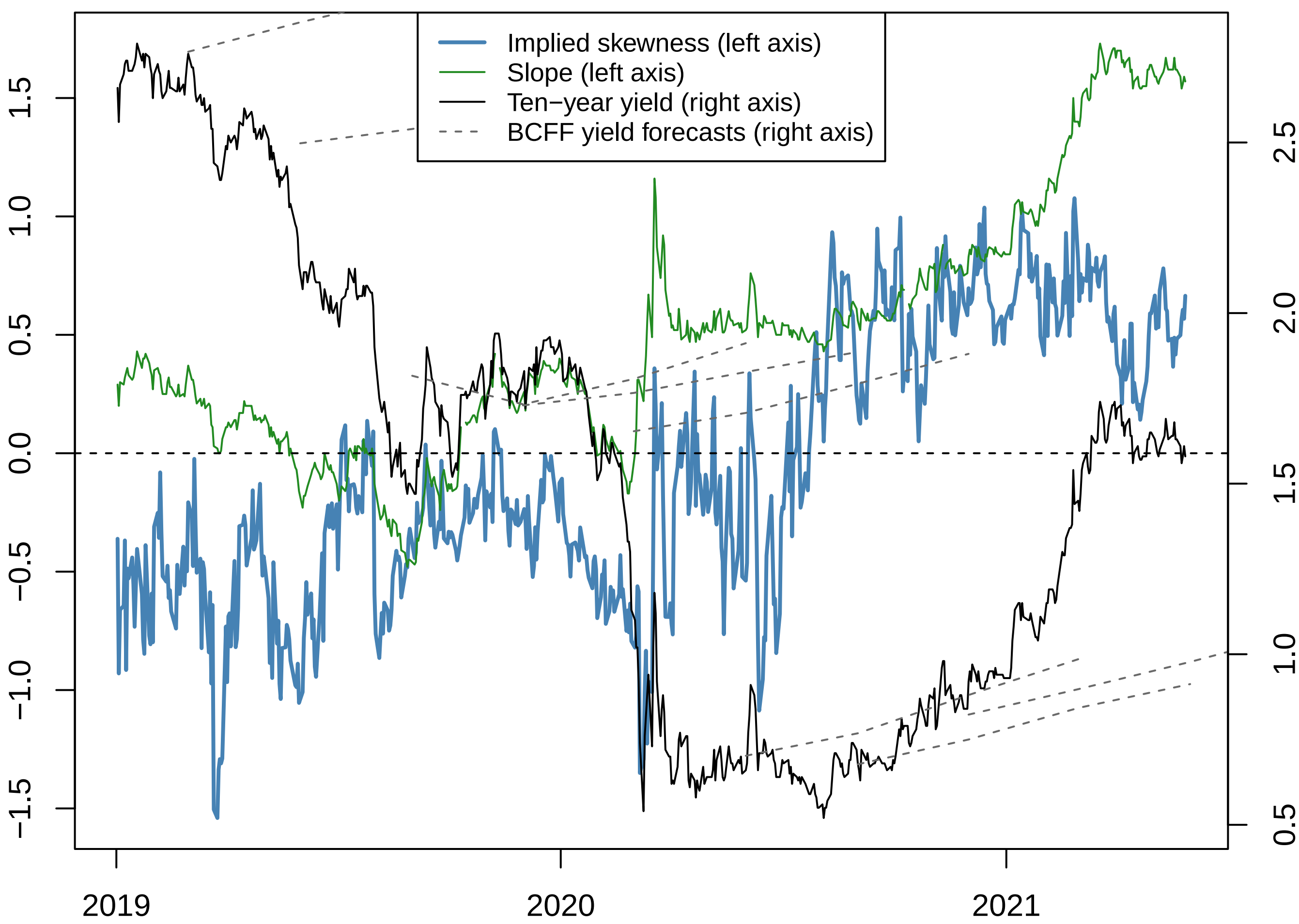

Figure 5: **Bias in beliefs and skewness.** Sample skewness of interest rates simulated from a simple model with biased beliefs (described in Appendix A.5). For each value of Δ, 10,000 paths of the real short rate are simulated over a ten-year horizon, using monthly time increments, and the skewness coefficient is calculated for the distribution of the final value of the short rate. Parameter settings are $x = 0.04$, $\gamma = 5$, $\sigma = 0.02$.

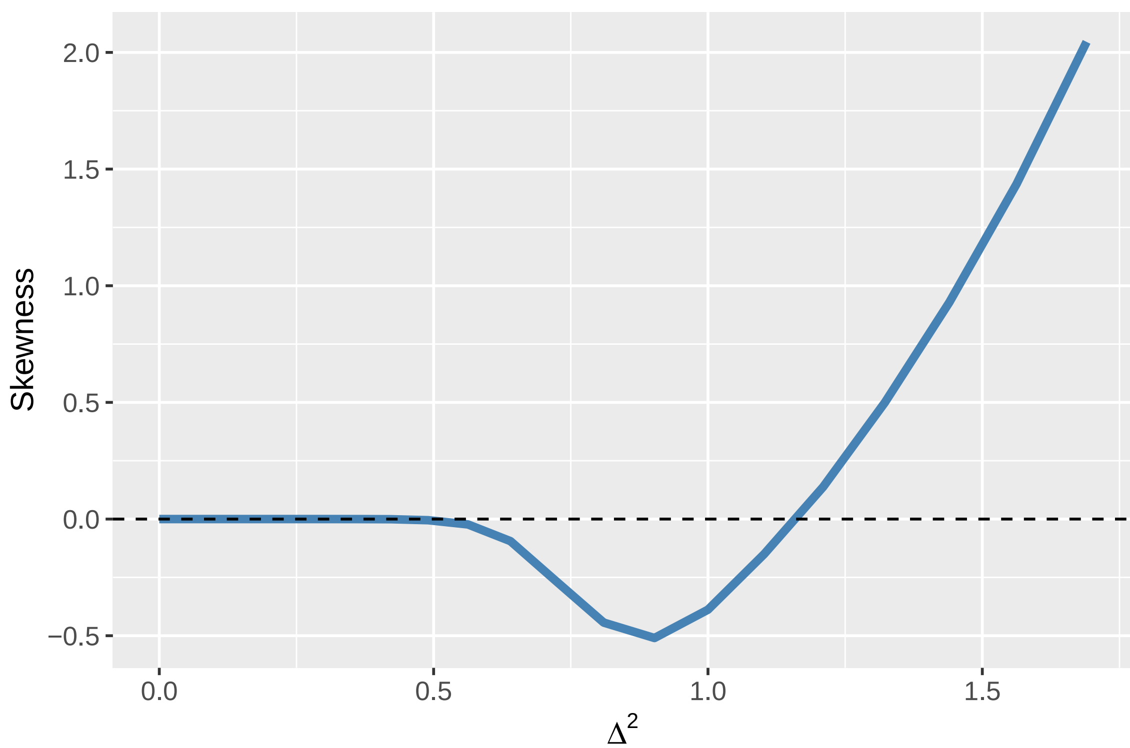

Table 1: Summary statistics for changes in 10-year Treasury yield

	Full sample		1990-2004		2005-2021	
Summary statistics of quarterly yield changes						
Mean	-0.05	(-0.13, 0.01)	-0.07	(-0.18, 0.03)	-0.04	(-0.14, 0.05)
Median	-0.02	(-0.10, 0.04)	-0.04	(-0.18, 0.13)	-0.02	(-0.14, 0.06)
Variance	0.23	(0.18, 0.28)	0.23	(0.18, 0.29)	0.23	(0.16, 0.31)
Third moment	-0.03	(-0.06, 0.01)	0.00	(-0.04, 0.04)	-0.05	(-0.11, 0.01)
Skewness of m-month yield changes						
$m = 1$	0.03	(-0.41, 0.52)	0.52	(0.12, 0.96)	-0.49	(-1.20, 0.16)
$m = 2$	-0.41	(-1.27, 0.36)	0.46	(0.15, 0.79)	-1.05	(-2.52, -0.20)
$m = 3$	-0.24	(-0.56, 0.05)	0.01	(-0.35, 0.34)	-0.46	(-1.05, -0.02)
$m = 6$	0.03	(-0.30, 0.37)	0.27	(-0.20, 0.77)	-0.27	(-0.71, 0.19)
$m = 12$	0.36	(-0.10, 0.84)	0.40	(-0.30, 1.10)	0.37	(-0.19, 1.17)
Skewness of m-month (negative) futures price changes						
$m = 1$	-0.14	(-0.63, 0.41)	0.40	(0.02, 0.81)	-0.69	(-1.47, 0.04)
$m = 2$	-0.20	(-0.66, 0.30)	0.32	(0.05, 0.61)	-0.72	(-1.55, -0.01)
$m = 3$	-0.16	(-0.39, 0.06)	0.15	(-0.07, 0.40)	-0.45	(-0.84, -0.14)

Summary statistics for changes in 10-year Treasury yield and futures prices. Sample period: January 1990 to May 2021.

Table 2: Predicting realized skewness

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
RSK	0.43***		0.20***		0.40***		0.19***
	(0.04)		(0.06)		(0.05)		(0.06)
ISK		2.05***	1.51***			2.04***	1.52***
		(0.22)	(0.27)			(0.23)	(0.27)
Level				−0.02	−0.02	−0.02	−0.02
				(0.06)	(0.04)	(0.04)	(0.03)
Slope				0.27***	0.14**	0.05	0.04
				(0.08)	(0.06)	(0.06)	(0.06)
Curvature				0.08	0.15	−0.48	−0.31
				(0.55)	(0.33)	(0.34)	(0.31)
Constant	0.08	−0.09	−0.06	−0.39	−0.18	0.07	0.06
	(0.07)	(0.08)	(0.07)	(0.47)	(0.32)	(0.29)	(0.26)
Observations	376	376	376	376	376	376	376
R^2	0.18	0.23	0.26	0.05	0.19	0.24	0.26

Predictive regressions for one-month realized skewness (RSK). *ISK* is option-implied yield skewness; *RSK* is realized yield skewness based on daily changes in futures prices and implied volatilities, following Neuberger (2012); *Level*, *Slope* and *Curvature* are the first three principal components of Treasury yields from one to ten years maturity (appropriately scaled). All predictors are measured at the end of the previous month. Sample: monthly observations from January 1990 to May 2021. Newey-West standard errors with automatic bandwidth selection are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table 3: Explaining the level of conditional yield skewness

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
RSK	0.14***		0.13***		0.13***		0.11***
	(0.02)		(0.02)		(0.02)		(0.01)
Level		0.004	0.004	0.06	0.06***		0.01
		(0.02)	(0.01)	(0.04)	(0.02)		(0.01)
Slope		0.10***	0.06***	0.20***	0.17***		−0.01
		(0.03)	(0.02)	(0.06)	(0.04)		(0.03)
Level*Slope				−0.03**	−0.03***		
				(0.01)	(0.01)		
Easing						0.23***	0.19***
						(0.08)	(0.07)
Tightening						−0.16**	−0.10*
						(0.08)	(0.06)
Constant	0.10***	−0.11	−0.05	−0.32	−0.27**	0.07	0.03
	(0.03)	(0.14)	(0.10)	(0.19)	(0.12)	(0.06)	(0.11)
Observations	377	377	377	377	377	377	377
R^2	0.44	0.15	0.50	0.19	0.54	0.31	0.58

Regressions for the level of option-implied yield skewness of the ten-year Treasury yield, using monthly data from January 1990 to May 2021. *RSK* is monthly realized yield skewness based on daily changes in futures prices and implied volatilities, following Neuberger (2012); *Level* and *Slope* are the first two principal components of Treasury yields from one to ten years maturity, scaled to correspond to level and slope of the yield curve; *Easing* and *Tightening* are dummy variables indicating whether the Federal Reserve was easing or tightening monetary policy one year ago (based on observed changes in the policy rate). Newey-West standard errors (with automatic bandwidth selection) are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table 4: Predicting excess returns

	(1)	(2)	(3)	(4)	(5)	(6)
Level	0.003**	0.004**	0.004**	0.004**	0.01***	0.01***
	(0.002)	(0.002)	(0.001)	(0.002)	(0.01)	(0.002)
Slope	0.02*	0.03***	0.03**	0.03***	0.05***	0.05***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Curvature	0.03	0.004	0.04	0.01	−0.05	−0.02
	(0.06)	(0.06)	(0.06)	(0.06)	(0.07)	(0.06)
ISK		−0.34***		−0.28**	−0.32***	−0.29**
		(0.12)		(0.13)	(0.12)	(0.14)
RSK			−0.06***	−0.02		
			(0.02)	(0.02)		
i^*					−0.26**	
					(0.12)	
GLS						−0.39**
						(0.17)
Constant	−0.07	−0.15*	−0.11	−0.15	0.26	−0.03
	(0.10)	(0.09)	(0.09)	(0.09)	(0.18)	(0.14)
Observations	374	374	374	374	374	347
R^2	0.06	0.11	0.09	0.11	0.13	0.14

Predictive regressions for three-month excess bond returns (average of duration-normalized excess returns on Treasury bonds with one to ten years maturity) using monthly data from January 1990 to May 2021. Predictors: *Level*, *Slope* and *Curvature* are the first three principal components of end-of-month Treasury yields from one to ten years maturity (appropriately scaled); *ISK* is option-implied yield skewness averaged over the last five business days of the month; *RSK* is monthly realized yield skewness based on daily changes in futures prices and implied volatilities, following Neuberger (2012); i^* is an estimate of the trend component of nominal interest rates from Bauer and Rudebusch (2020); *GLS* is survey disagreement about future ten-year yields from Giacoletti, Laursen, and Singleton (2021). Standard errors based on the reverse regression delta method of Wei and Wright (2013) are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table 5: Predicting FOMC surprises with ISK and RSK

	(1)	(2)	(3)	(4)	(5)
Level	−0.0002		−0.001	−0.001	−0.001
	(0.001)		(0.001)	(0.001)	(0.001)
Slope	−0.001		−0.007***	−0.003	−0.007***
	(0.002)		(0.003)	(0.002)	(0.003)
Curvature	−0.016		−0.025*	−0.015	−0.029**
	(0.013)		(0.014)	(0.013)	(0.014)
ISK		0.030***	0.043***		0.059***
		(0.009)	(0.012)		(0.016)
RSK				0.004**	−0.005*
				(0.002)	(0.003)
Constant	0.003	−0.010***	0.018**	0.008	0.017*
	(0.007)	(0.003)	(0.009)	(0.008)	(0.009)
Observations	213	213	213	213	213
R^2	0.009	0.056	0.099	0.027	0.112

Predictive regressions for the monetary policy surprise around FOMC announcements from January 1994 to June 2019. The dependent variable is the first principal component of 30-minute futures rate changes around the announcement for five different contracts with up to about one year maturity. *Level*, *Slope* and *Curvature* are the first three principal components of Treasury yields from one to ten years maturity (appropriately scaled) measured on the day before the announcement; *ISK* is option-implied yield skewness, *RSK* is realized yield skewness based on daily changes in futures prices and implied volatilities, following Neuberger (2012), both implied and realized skewness use data over the month (22 trading days) before the FOMC announcement. White heteroskedasticity-robust standard errors are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table 6: Predicting FOMC surprises with ISK and macro variables

	(1)	(2)	(3)	(4)
ISK	0.031***	0.028***	0.032***	0.021***
	(0.009)	(0.009)	(0.009)	(0.008)
FFR	−0.001			
	(0.001)			
Annual employment growth	0.455***			
	(0.169)			
BBK index		0.010**		
		(0.004)		
Change in employment			0.057***	
			(0.018)	
S&P 500 return				0.135***
				(0.049)
Constant	−0.013***	−0.008***	−0.016***	−0.011***
	(0.004)	(0.003)	(0.004)	(0.003)
Observations	213	213	213	213
R^2	0.084	0.107	0.130	0.128

Predictive regressions for the monetary policy surprise around FOMC announcements from January 1994 to June 2019. The dependent variable is the first principal component of 30-minute futures rate changes around the announcement for five different contracts with up to about one year maturity. *ISK* is option-implied yield skewness averaged over the month (22 trading days) before the FOMC announcement; *FFR* is the average federal funds rate over the calendar month preceding the meeting, and *Annual employment growth* is the 12-month log-change in total nonfarm payroll employment (appropriately lagged), as used by Cieslak (2018); *BBK index* is the Brave-Butters-Kelley business cycle indicator form the Chicago Fed, *Change in employment* is the change in non-farm payrolls released in the most recent employment report, and *S&P 500 return* is the stock return over the three months (65 days) up to the day before the FOMC announcement, as used by Bauer and Swanson (2020). White heteroskedasticity-robust standard errors are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table 7: Predicting Blue Chip forecast errors

	Current	1Q ahead	2Q ahead	3Q ahead	4Q ahead	5Q ahead
	(1)	(2)	(3)	(4)	(5)	(6)
ISK	0.13***	0.33***	0.37**	0.43**	0.58***	0.70***
	(0.04)	(0.10)	(0.15)	(0.17)	(0.18)	(0.17)
Level	0.004	0.01	0.01	0.02	0.03	−0.02
	(0.01)	(0.02)	(0.04)	(0.04)	(0.04)	(0.07)
Slope	−0.005	−0.05	−0.09	−0.15*	−0.20*	−0.31**
	(0.02)	(0.04)	(0.06)	(0.09)	(0.10)	(0.13)
Curvature	−0.01	0.14	0.32	0.35	0.39	0.29
	(0.10)	(0.22)	(0.27)	(0.39)	(0.50)	(0.48)
Constant	−0.07	−0.21	−0.35*	−0.45	−0.56	−0.40
	(0.05)	(0.13)	(0.19)	(0.28)	(0.39)	(0.56)
Observations	372	371	368	365	362	276
R^2	0.03	0.05	0.06	0.07	0.10	0.18

Predictive regressions for forecast errors in the h-quarter ahead consensus Blue Chip Financial Forecasts for the ten-year Treasury yield, using monthly surveys from January 1990 to January 2021. The forecast horizon h ranges from 0 (current/nowcast) to 5. *ISK* is option-implied yield skewness; *Level*, *Slope* and *Curvature* are the first three principal components of end-of-month Treasury yields from one to ten years maturity (appropriately scaled). All predictors are measured on the day of the survey deadline. Hansen-Hodrick standard errors with $3(h+1)$ lags are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table 8: Inflation and yield forecast errors in the Blue Chip survey

	Dependent variable: Blue Chip inflation forecast error					
	Current	1Q ahead	2Q ahead	3Q ahead	4Q ahead	5Q ahead
	(1)	(2)	(3)	(4)	(5)	(6)
Yield forecast error	1.31**	1.00**	0.56**	0.36**	0.33*	0.67**
	(0.57)	(0.45)	(0.25)	(0.16)	(0.17)	(0.28)
Constant	0.09	0.16	0.08	−0.04	−0.07	0.44
	(0.11)	(0.17)	(0.17)	(0.18)	(0.22)	(0.33)
Observations	372	371	368	365	362	276
R^2	0.05	0.07	0.04	0.02	0.02	0.06

Regressions of forecast errors in the h-quarter-ahead consensus Blue Chip Financial Forecasts for CPI inflation on corresponding forecast errors for the ten-year Treasury yield, using monthly surveys from January 1990 to May 2021. The forecast horizon h ranges from 0 (current/nowcast) to 5. Hansen-Hodrick standard errors with $3(h+1)$ lags are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table A1: Predicting excess returns: pre-ZLB sample

	(1)	(2)	(3)	(4)	(5)	(6)
Level	0.01**	0.01***	0.01**	0.01***	0.03**	0.01***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.01)	(0.004)
Slope	0.03*	0.03**	0.03**	0.03**	0.09**	0.05***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)
Curvature	0.01	0.09	0.05	0.09	−0.06	0.03
	(0.12)	(0.11)	(0.11)	(0.11)	(0.18)	(0.11)
ISK		−0.75***		−0.69***	−0.73***	−0.64***
		(0.24)		(0.26)	(0.23)	(0.23)
RSK			−0.09**	−0.02		
			(0.04)	(0.04)		
i^*					−0.56*	
					(0.30)	
GLS						−0.52*
						(0.27)
Constant	−0.33	−0.27	−0.36	−0.28	0.34	−0.17
	(0.25)	(0.26)	(0.24)	(0.26)	(0.34)	(0.24)
Observations	227	227	227	227	227	227
R^2	0.06	0.14	0.10	0.14	0.20	0.17

Predictive regressions for three-month excess bond returns (average of duration-normalized excess returns on Treasury bonds with one to ten years maturity) using monthly data from January 1990 to November 2008. Predictors: *Level*, *Slope* and *Curvature* are the first three principal components of end-of-month Treasury yields from one to ten years maturity (appropriately scaled); *ISK* is option-implied yield skewness averaged over the last five business days of the month; *RSK* is monthly realized yield skewness based on daily changes in futures prices and implied volatilities, following Neuberger (2012); i^* is an estimate of the trend component of nominal interest rates from Bauer and Rudebusch (2020); *GLS* is survey disagreement about future ten-year yields from Giacoletti, Laursen, and Singleton (2021). Reverse regression standard errors, using the reverse regression delta method of Wei and Wright (2013), are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table A2: Predicting target and path FOMC surprises

	Dependent variable:									
	Target surprise					Path surprise				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Level	−0.003					0.002				
	(0.002)					(0.002)				
Slope	−0.005					−0.01***				
	(0.004)					(0.005)				
Curvature	−0.06**					−0.01				
	(0.03)					(0.02)				
ISK	0.09***	0.03*	0.03*	0.04**	0.02	0.05**	0.04***	0.03**	0.04***	0.03**
	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
RSK	−0.01***					0.003				
	(0.01)					(0.004)				
FFR		−0.004*					0.002			
		(0.002)					(0.002)			
Empl. growth		0.42*					0.65**			
		(0.24)					(0.31)			
BBK index			0.01					0.02***		
			(0.01)					(0.01)		
Change in empl.				0.06**					0.08***	
				(0.03)					(0.03)	
S&P 500 return					0.17**					0.14*
					(0.08)					(0.08)
Constant	0.04**	0.002	−0.002	−0.01	−0.004	0.02	−0.02**	−0.001	−0.01**	−0.004
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.004)	(0.01)	(0.004)
Observations	213	213	213	213	213	213	213	213	213	213
R^2	0.09	0.04	0.04	0.05	0.06	0.08	0.08	0.08	0.09	0.07

Predictive regressions for the target and path factor of the monetary policy surprise around FOMC announcements from January 1994 to June 2019, measured as in Gürkaynak, Sack, and Swanson (2005). For a description of the predictors see the notes to Tables 5 and 6. White heteroskedasticity-robust standard errors are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table A3: Predicting SPF forecast errors

	Dependent variable: SPF forecast error									
	Current		1Q ahead		2Q ahead		3Q ahead		4Q ahead	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ISK	0.16***	0.19***	0.48***	0.56***	0.57***	0.69***	0.57***	0.75***	0.59*	0.83***
	(0.04)	(0.04)	(0.12)	(0.10)	(0.18)	(0.17)	(0.19)	(0.26)	(0.33)	(0.29)
Level		0.002		−0.003		−0.01		0.005		0.02
		(0.01)		(0.03)		(0.03)		(0.06)		(0.06)
Slope		−0.01		−0.05*		−0.10**		−0.12		−0.16
		(0.01)		(0.03)		(0.05)		(0.08)		(0.10)
Curve		−0.17**		−0.13		0.04		−0.04		0.02
		(0.09)		(0.17)		(0.26)		(0.38)		(0.46)
Constant	−0.08***	−0.01	−0.25***	−0.09	−0.40***	−0.20	−0.54***	−0.30	−0.69***	−0.45
	(0.02)	(0.04)	(0.06)	(0.10)	(0.08)	(0.15)	(0.07)	(0.27)	(0.14)	(0.38)
Observations	118	118	117	117	116	116	115	115	114	114
R^2	0.09	0.14	0.08	0.10	0.07	0.09	0.05	0.08	0.04	0.09

Predictive regressions for forecast errors in the h-quarter ahead median forecast for the ten-year Treasury yield in the Survey of Professional Forecasters, using quarterly surveys from 1992:Q1 to 2020:Q3. The forecast horizon h ranges from 0 (current/nowcast) to 4. *ISK* is option-implied yield skewness, *Level*, *Slope* and *Curvature* are the first three principal components of Treasury yields from one to ten years maturity (appropriately scaled), measured on the day before the survey deadline. All predictors are measured on the day before the survey deadline. Hansen-Hodrick standard errors with h lags are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Table A4: Inflation and yield forecast errors in the SPF

	Dependent variable: SPF inflation forecast error				
	Current	1Q ahead	2Q ahead	3Q ahead	4Q ahead
	(1)	(2)	(3)	(4)	(5)
Yield forecast error	1.60	1.10***	0.52*	0.28	0.34*
	(1.21)	(0.40)	(0.29)	(0.19)	(0.20)
Constant	0.13	0.17	0.06	−0.05	−0.02
	(0.11)	(0.15)	(0.18)	(0.18)	(0.22)
Observations	115	114	113	112	111
R^2	0.05	0.08	0.03	0.01	0.02

Regressions of forecast errors in the h-quarter-ahead median forecast for CPI inflation in the Survey of Professional Forecasters on the corresponding forecast errors for the ten-year Treasury yield, using quarterly surveys from 1992:Q1 to 2020:Q3. The forecast horizon h ranges from 0 (current/nowcast) to 4. Hansen-Hodrick standard errors with h lags are reported in parentheses, and *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.